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Uncertainty in power system planning

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by

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ACRONYMS AND ABBREVIATIONS

R2B	Resource to backbone transmission
x^f	Flexible trajectory
GENCO	Generating Companies
SP	Stochastic Programming
TEP	Transmission Expansion Planning
GEP	Generation Expansion Planning
CEP	Co-optimization Expansion Planning
MILP	Mixed Integer Linear Programs
Matlab	Matrix Laboratory
AC	Adaptation Cost
CF	Capacity Factor
WF	Wind farms
CC	Capacity Credit
RPS	Renewable Portfolio Standard
GS	Global Scenario
LDC	Load Duration Curve
RO	Robust Optimization
DG	Distributed generation
O&M	Operation and Maintenance
LSE	Load Serving Utility
GHG	Greenhouse Gas
LP	Linear Program

FC	Fuel costs
ISU	Iowa State University
ECPE	Electrical and Computer engineering
NG	Natural Gas
AWEA	American Wind Energy Association
B&B	Branch and Bound
ATC	Available Transfer Capability
PTC	Production tax credit
DG	Distributed generation
MW	Mega-Watts
NP-Hard	Non-deterministic Polynomial-time hard

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ABSTRACT

Deterministic transmission planning is based on deterministic assumptions, by planning for a single forecasted set of conditions. The issue of adaptability in transmission and generation expansion planning has become important for planners in order to deal with future uncertainties. Adaptability is “the ability to change (or be changed) to fit changed circumstances” [1]. Adaptability planning helps mitigate losses in unforeseen situations and exploit opportunities in expected situations. Adaptability also provides the opportunities to take alternative actions after events unfold. Adaptable plans can help reduce future costs and time, and it also reduces complexity associated with possible future changes to the existing system. In order to assess the value of adaptable planning in this dissertation, our approach is illustrated using the IEEE 24 bus system and the Iowa system. Our adaptable planning shows there is benefit in incorporating uncertainty into power system planning when compared to deterministic planning which is the conventional approach.

CHAPTER 1. INTRODUCTION

1.1 Introduction To Power System Planning

Modern-day synchronized electric power grids can be regarded as very large machines. As the electric grid continues to evolve, planning and implementation of new investment is an issue power system planners have to consider. Power system planning is a very difficult and comprehensive process because it involves projecting into the future and making investment decisions that will satisfy demand growth, reliability and other constraints that ensure the satisfactory performance of a power system. The integration of renewables in to the electric power grid has also become a challenge for power system planners because renewables are not dispatched conventionally like other generating technologies and could cause reliability issues if the impact of renewables is not properly considered.

Power system planning can be performed for short, medium and long-term periods. Power system planning can also be performed at the distribution, transmission or generation levels. The decision power system planners make are where to locate new investment, the capacity of new investment, the timing of new investment, and what type of technology should be installed. One of the most important tasks to be done before planning is performed is load forecasting. The electric load is forecasted for the entire planning period; planning constraints ensure that the supply of generation will be adequate throughout the planning horizon. Factors determining load forecasting are expected population growth rate and other economic indices like Gross Domestic Product. Power system planning is a very computational task due to the fact that an interconnected power systems involved a very large number of components,

- each requiring representation, including existing lines, existing generation, projected retirements, load forecasts, candidate generation technologies, candidate transmission line technologies, and uncertainties, and
- each requiring operational coordination within the system, according to the physical laws governing electric power flow (e.g., nodal balance and impedance effects)

The objective of a power system planning problem is to minimize investment costs and cost of generation for the planning horizon. Power system planning can be either static or dynamic. Static planning is when planning is done for single stage or period while dynamic planning is done for multiple stages and provides a solution for multiple stages in a single formulation.

1.2 Motivation

The major motivating factor for this dissertation is that power systems are necessarily planned under uncertainty, since the planning period is always in the future. Change is inevitable and power system planners have realized that planning based deterministic futures can have serious economic and reliability consequences if the future turns out to differ from the one that was assumed when the plan was developed as it inevitably does. The next section discusses the types of uncertainties power system planners have to consider and how uncertainty is modelled in this research.

Power system planning engineers understand that planning on a single future is unreasonable. Uncertainty in fuel cost, government policies, demand, technology change and investment costs are inherent to the power system planning problem. For example, fuel price is highly uncertain, especially for natural gas, both in the short-term where it incurs high volatility, and in the long-term, where its price is heavily dependent on the uncertainties of gas supply and demand. Government policies, such as renewable portfolio standards (RPS), production tax credits, and greenhouse gas penalties, are also uncertain and must be faced by power system planners. RPS is a regulation introduced by different states to increase energy production from non-conventional sources such as wind, solar, geothermal and biomass. California has mandated that by 2020, 33% of electricity generation should come from renewable generation [2]. The production tax credit is an incentive initiated by the federal government to subsidize the production of electricity from renewables; it is typically renewed for between one and three years after which it may or may not be in place.

One of the challenges the power industry is facing is climate change. This has put heavy pressure on generating companies (GENCO'S) to retire many of the coal generating plants. Government policies on carbon dioxide (CO_2) is also another source of uncertainty. The idea of the carbon tax is to penalize each ton of Greenhouse Gas (GHG) emitted. So far, there is no penalty associated with greenhouse gasses (e.g., carbon dioxide); however, it is certainly possible, and perhaps even likely, that such a cost will be imposed within the next ten years, a time frame which is well within the planning horizons of most electric generation and transmission owners. Nonetheless, the US federal government has been utilizing other ways to encourage utilities to shift from fossil-based technologies to cleaner technologies such as renewable energy. For example, the US Environmental Protection Agency (EPA) has imposed rules called the Mercury/Air Toxics Standards (MATS) which limits the amount of hazardous air pollutants that can be emitted from power plants. A more recent EPA-sourced ruling is referred to as 111d which would limit the amount of CO_2 emitted from each state, forcing the retirement of many coal plants. However, this ruling has been challenged in the courts, and at the time of this writing, it is uncertain whether it will be enforced or not. These policy-related influences have led to the retirement of coal- and oil-fired technologies, discouraging investment in these technologies.

Another uncertainty that is having heavy influence on power system planning is the extent to which distributed generation will play a role in the future resource portfolio. Distributed generation refers to generation sited at or close to the point of use, typically interconnected at the distribution (or low) voltage levels. As distribution generation penetration increases, there will be an increase in the uncertainty associated with building conventional generation and transmission.

Power system planning also faces uncertainties such as unforeseen scientific breakthroughs in transmission and generation technologies, i.e., in the maturation rates of technologies that can be planned. For example, extracting energy from the ocean tides, although possible, is very expensive today. As long as the investment cost of tidal energy remains at its current level, it will not play a significant role in future planning alternatives. However, if the investment cost of tidal energy were to dramatically drop due to a one or more technological developments which significantly decrease its investment cost, tidal energy could become a major player along the Atlantic, Gulf, and Pacific coasts of the US. This would be extremely significant because the U.S. load centers are largely located in these areas. Whether tidal energy investment cost actually decreases in this fashion is highly uncertain.

If present plans do take into account all these uncertainties, the plan may cause unforeseen economic consequences or even become obsolete if drastic changes occur in the future. Figure 1, below, illustrates the various uncertainties typically considered by power system planners. The major problem for handling these uncertainties today within the power system planning function is that there are few methods, if any, which are computationally tractable for doing this. In this dissertation, we aim to develop one such method. Uncertainty is modelled using what we call global scenarios. The uncertainty is represented as a specified set of trajectories through the time intervals, one for each defined "future," where each trajectory represents a set of realizations on global uncertainties at each stage or time interval.

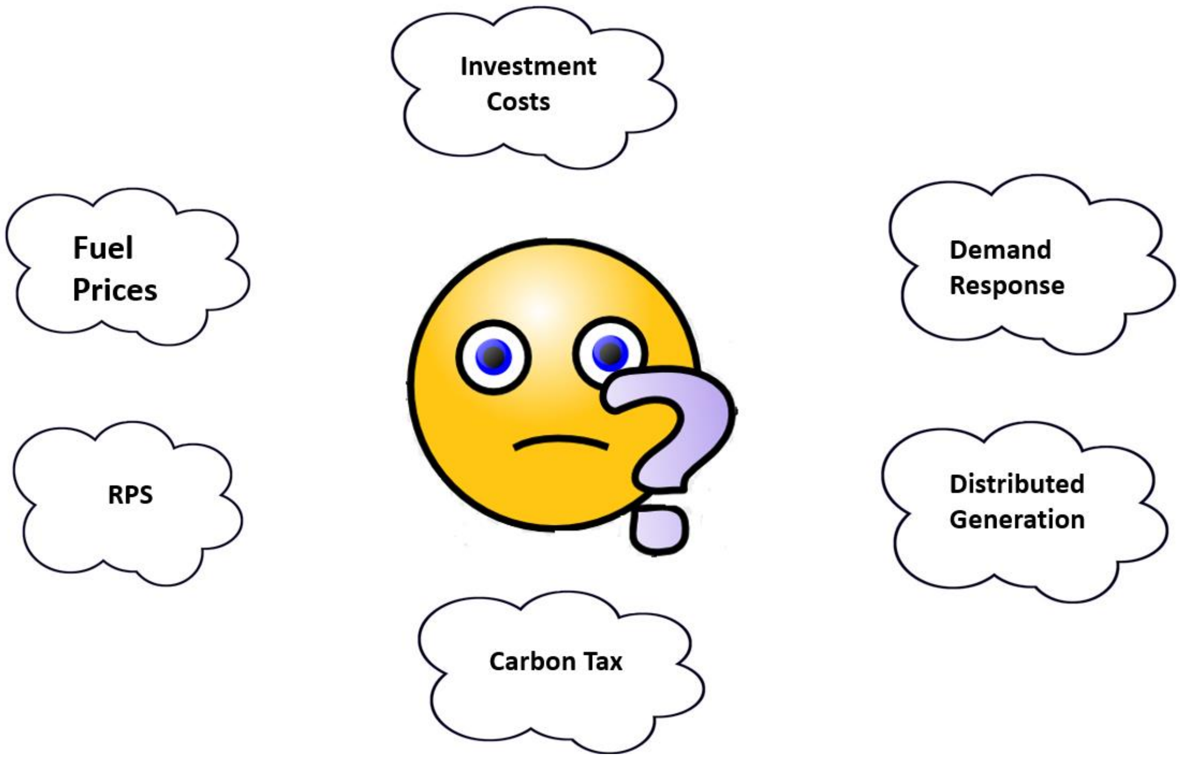

Figure 1: Uncertainties faced by power system planners

1.3 Problem Statement

The problem addressed in this work is that transmission and generation take years to build and therefore are necessarily planned without knowing the future conditions to which they will be exposed. In addition, they are capital-intensive. We therefore seek to identify transmission and generation investment plans that are effective in adapting to a variety of possible futures. We use multi-period optimization for transmission only and for simultaneous selection of generation and transmission (co-optimization), with investments distinguished between core investments (what is planned to be built) and adapted investments (what will need to be built once a particular future is revealed).

1.4 Application: Resource To Backbone Transmission

Wind power has been growing at a very fast rate in the United States. The state of Iowa is also in the forefront of wind generation. According to the American Wind Energy Association "Iowa led the nation by producing 28.5 percent of its electricity from wind power, followed by South Dakota at 25.3 percent and Kansas at 21.7" [3]. The retirement of coal plants due to stringent policies on CO2 emissions will continue to aid wind growth. The production tax credit (PTC) has also aided the growth of wind power investment. The production tax credit is an incentive given by the federal government for the production of renewable energy. Advances in wind turbine technology have also led to the increase in wind power growth. Despite the fact there has been significant increase in wind power and wind power projected to supply a significant amount of electric energy required by the U.S., the transmission to transfer this power to load centers is insufficient. Most wind farms tend to be located in remote areas where there is not enough available transmission capacity to transfer most of this power to where they are needed. Another issue is that sometimes available substations to connect most of this wind farms are very far from where this wind farms are located.

Therefore, one kind of transmission design that has recently become of interest in the industry, and, like any other transmission design, is subject to uncertainty, is the so-called "resource to backbone" (R2B) transmission. This transmission design problem is motivated by the possibility that wind energy penetration will increase significantly over the next 40 years in certain regions typically relatively remote from load centers, e.g., the Midwest U.S. and in particular, Iowa, and that high capacity backbone transmission will be needed to transfer much of this energy to the eastern part of the U.S. where the major load centers are located. One transmission design problem is the design of the so-called “backbone transmission” to perform this long-distance power transfer. A subsequent and related design problem is the design of the transmission necessary to move power from the windfarms up onto the backbone transmission, the problem we denote as the R2B transmission design problem.

We identify the backbone transmission design problem as the “level 3” problem of moving wind energy; typically, transmission voltages considered in this problem include 345, 500, or 765 kV AC, and 500, 600, or 800 kV DC. The R2B problem is the “level 2” problem; typically, transmission voltages considered in this problem include 69, 138, 161, and 230kV AC. We classify the familiar (to the industry) problem of collection within a windfarm as the “level 1” problem; typically, the 34.5 kV AC distribution voltage is utilized at this level. These three problems are illustrated in Fig. 2 below. The “level 2” R2B problem in which we have particular interest in this dissertation is appropriately characterized as one in which a multi-farm collection network must be designed.

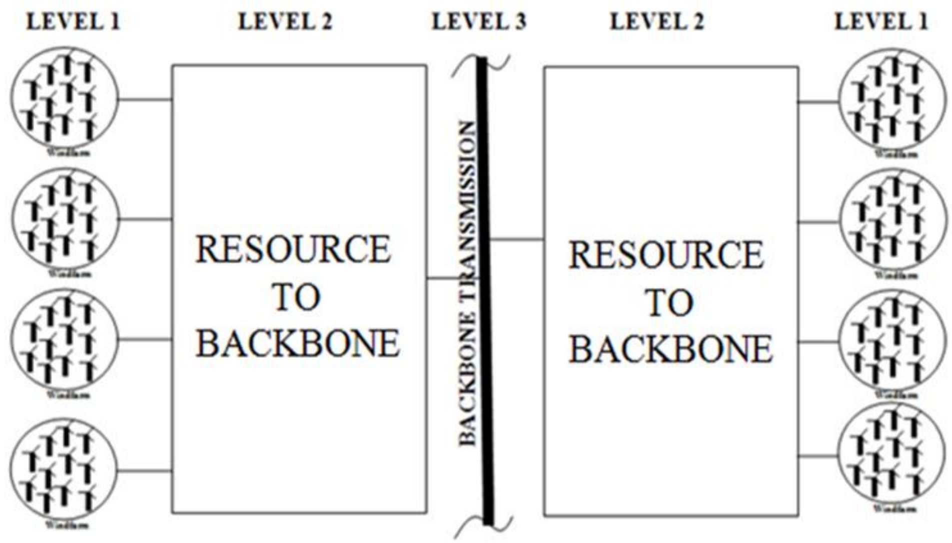

Figure 2: A 3-level conceptualization

Thus, we will apply our computational methods of planning under uncertainty to the problem of cost-effective design of R2B transmission so as to maximize future flexibility. Thus, in this research; we explore and develop analytical and qualitative approach in designing flexible R2B transmission networks.

1.5 Contributions

The contributions of this thesis are described below.

- **Extended the adaptation for generation expansion planning to transmission expansion planning**: When it comes to consider global uncertainties in planning, it is expensive constructing a robust infrastructure that is able to perform well under all of the different futures. This motivated the desire to design transmission systems that are adaptable under global uncertainties.

- **Extended the adaptation for generation expansion planning to co-optimization expansion planning**: A co-optimization formulation under uncertainty will help power system planners effectively co-ordinate the generation and transmission planning under uncertainty.
- **Development of a scenario reduction technique for both transmission planning and co-optimization planning**: Scenario reduction can make large-scale transmission and generation computationally tractable.
- **Identification of the relationship of stochastic programming to adaptation**: The conceptual similarities and differences are highlighted, and formulational similarities and differences.
- **Designed R2B transmission design under uncertainty for the state of Iowa**: Developed procedures for designing R2B transmission under uncertainty and applied it to the Iowa power system. A backbone is designed in order to increase the available transfer capability within and out of the state of Iowa.

1.6 Thesis Organization

This section gives a general summary of the dissertation. This dissertation is divided into 7 chapters. The order, format and contents of these chapters are described below.

Chapter 2

Chapter two provides a literature review related to this dissertation. The purpose of this chapter is to analyze previously published work related to the dissertation objective in terms their strengths and weaknesses. This chapter also describes the rationale for proposing a new approach instead of using previously known approaches. This chapter describes known approaches used for decision making under uncertainty and the proposed approach used in this work. Another major section in this chapter is description of how previous approaches have been used to solve problems faced in the power industry.

Chapter 3

Chapter three compares and contrasts the proposed approach used in this dissertation and a widely used approach in decision making under uncertainty (i.e. Stochastic Programming). The formulations for both approaches are described and explained. The conceptual foundation for both approaches are described.

Chapter 4

Chapter 4 describes mathematical models for adaptation approach. The mathematical formulation for the adaptation is formulated and described. Scenario reduction technique used is also described.

Chapter 5

This chapter illustrates the application of adaptation to transmission expansion planning using a test system – the IEEE 24 bus test system. Uncertainty modeling and scenario reduction are illustrated. This chapter also illustrates illustrates the application of adaptation to co-optimization expansion planning using IEEE 24 bus system. In this co-optimization formulation the transmission candidates decision variables are integer while the generation decision variables are continuous variables. This chapter also shows the benefit of incorporating uncertainty into the decision making process of power system planning.

Chapter 6

This chapter describes mathematical models used to design R2B transmission under uncertainty. This chapter also describes the Iowa power system used in this work and a backbone transmission is designed that would transfer most of this wind to states eastward from Iowa where the load is high. The transmission designs obtained are analyzed in terms their strengths and weaknesses. The adaptive designs are also compared to deterministic designs, and the benefits of including uncertainty into planning is analyzed. The computational techniques applied to solve the models are described. The scenario reduction technique as it is applied is also described.

Chapter 7

This chapter provides conclusion of this work and describes its significance, strengths, weakness and limitations. This chapter also identifies possible future continuations of this work. Recommendations based on findings in this research are also provided.

CHAPTER 2. LITERATURE REVIEW

There have been many approaches applied to power system planning and operations under uncertainty. This chapter gives an introduction to several well-known approaches used to solve power system problems when faced with uncertainty. This chapter also describes several previous papers that have used these approaches and how these approaches were deployed to solve the problem of uncertainty in the area of power system planning and operations. In this review we focus on decision theory, stochastic programming, robust optimization and real options as previous approaches and a new approach known as adaptation is also described, which is the approach used in this dissertation.

2.1 Decision Theory

Decision theory is the theory of decision making. Every decision maker is faced with the problem of how to make the best decisions. Decision theory helps a decision maker choose from a set of alternative based on their possible consequences and benefits. The main elements involved in decision making are alternatives, scenarios, consequences and criterion. Alternatives are possible choices in which one has to be chosen, a scenario is a possible future that can be characterized by factors such as economic, social, and technological factors, consequences are the outcomes of decisions that were made earlier, while criterion is the objective with which the decision maker uses to compare alternatives.

2.1.1 Decision making under uncertainty

This involves making decisions when the probabilities of future outcomes are unknown. There are different approaches to making decisions under uncertainty; they are Minimax, Minimin, Regret Minima, Hurwicz and Laplace criterion. The objective of any firm is to minimize revenues minus cost.

Minimax

The minimax principle computes the maximum costs for all alternatives and then finds the minimum of them all, the minimax principle seeks to minimize possible costs for worst case scenarios. Even though this approach is a very risk averse approach, it throws away too many information, hence focuses on extremes, this is considered pessimistic.

$$\Pi^* = \min_i \left(\max_j (P_{ij}) \right) \quad (2.1)$$

where P_{ij} is the profit of alternative i in scenario j

Regret Minimax

What Regret minimax does differently from the ordinary minimax approach is that it picks the best value from each scenario and subtracts it from the value of all other alternatives and then applies the minimax principle. Minimax regret selects the alternative that minimizes the maximum opportunity loss. That is it identifies the alternative with the objective π* according

$$\prod^* = \min_i (\max_j (R_{ij})) \quad (2.2)$$

where R_{ij} is the regret matrix

Laplace criterion

The Laplace criterion assumes that if the probabilities of different scenarios are not known, they should be assumed to be equal. This idea makes the Laplace approach similar to decision making under risk. The Laplace criterion applies the “principle of insufficient reason” by Jakob Bernoulli [4],which implies that if we are ignorant about the likelihood of events occurring in the future we have no reason to assume that one has a higher chance of occurring than the other. One who makes decision based on this criterion is considered a realist.

$$\Pi^* = \min_i \left(\frac{1}{n} \sum_{j=1}^{n} P_{ij} \right) \qquad (2.3)$$

where P_{ij} is the profit of alternative i in scenario j

Hurwicz criterion

This approach identifies best and worst case scenarios and combines them using alpha. When alpha is chosen close to 1, this implies that the decision maker is pessimistic about future, and when alpha is chosen to 0, this implies decision maker is optimistic about future. Alpha is between 0 and 1.

$$\Pi^* = \min_i \left(\alpha * \max_j (P_{ij}) - (1-\alpha) * \min_j (P_{ij}) \right) \qquad (2.4)$$

where P_{ij} is the profit of alternative i in scenario j

Minimin

A decision maker who makes decisions using this strategy is known as an optimist due to the fact that the decision maker looks for the best situation that could happen in each scenario and for all alternatives and chooses the alternative with the lowest value.

$$\Pi^* = \min_i (\min_j (\mathrm{P}_{ij})) \qquad (2.5)$$

where P_{ij} is the profit of alternative i in scenario j

2.1.2 Recent use of decision theory in power system application

Zhao et al. [5] defines a flexible plan as one that can adapt to future scenarios in a cost-effective and timely manner. Zhao introduces the concept of adaptation costs in order to assess flexibility. Zhao finds the optimal plan for each scenario and uses them as planning candidates. After all optimal candidate plans have been solved for each scenario, the selected flexible plan is one that minimizes the worst-case adaptation cost. Maghouli et al. [6] formulated a multi-stage multi-objective transmission expansion planning problem in which the three objectives used, were total social cost, robustness and flexibility. Heuristics were used to solve the mixed integer optimization problem. However, this method does not guarantee optimality. The regret minimax was used for decision making. The issue with regret minimax as described as Higle and Wallace is that "it is both pessimistic and sensitive to the choice of scenarios used when describing the problem"[7].

2.2 Stochastic Programming

Stochastic programming (SP) has been a conventional and wide spread approach for handling uncertainty. SP is based on the assumption that the probability of random data is known. This can be seen as a very strong assumption.

Stochastic optimization requires the following steps [8]:

1) Build a scenario tree
2) Assign probabilities to future outcomes
3) Optimize over all possibilities

2.2.1 Formulation

Two-stage stochastic optimization

The idea behind two-stage SP is that the decisions made should be based on information available now and not upon future realizations. This is a very common type of SP; it is formulated as follows:

$$\underset{x}{Minimize} \quad c^T x + E_\omega Q(\mathrm{x}, \omega) \tag{2.6}$$

s.t

$$\mathrm{Ax} = \mathrm{b} \tag{2.7}$$

$$\mathrm{x} \geq 0 \tag{2.8}$$

where,

$$Q(\mathrm{x}, \omega) = \min_{y} \; d_\omega^T y \tag{2.9}$$

$$T_\omega x + W_\omega y = h_\omega \tag{2.10}$$

$$y \geq 0 \tag{2.11}$$

x is the first stage variable and y is second stage variable, while represents different scenarios.

Multi-stage stochastic optimization

Two stage stochastic can be extended to multi-stage stochastic optimization. One of important properties of SP is called non-anticipativity. By non-anticipativity we mean that if two paths share the same history until a particular stage, they must also share the same decision until that particular stage. For instance in Fig. 3 below, path 7 and path 8 have the same history until stage 2; this means their solution must be the same until after stage 2. This can be interpreted that stochastic optimization can be viewed as a problem having the tree-like structure observed in

Fig. 3.

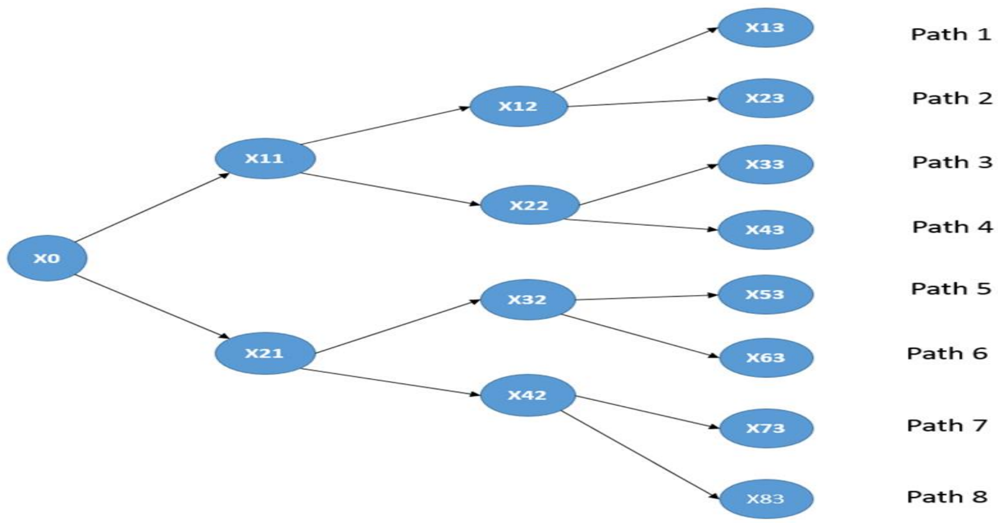

Figure 3: Tree-like structure of a stochastic programming problem

A multi-stage stochastic optimization with explicit non-anticipative constraints can be described as follows[9].

$$Min \sum_{s=1}^{S} P_s \sum_{t=1}^{T} c_{ts} x_{ts} \qquad (2.12)$$

Subject to

$$\sum_{t=1}^{t'} A_{t'ts} x_{ts} = b_{t's} \quad \forall s \in \{1......, S\}, t' \in \{1......, T\} \qquad (2.13)$$

$$l_{ts} \le x_{ts} \le u_{ts} \qquad \forall s \in \{1......, S\}, t \in \{1......, T\} \qquad (2.14)$$

where s stands for scenario, P_s is the probability of scenario s, t represent stages.

Non-anticipative constraints

Let n represent nodes in a scenario tree

$$x_{ts} = x_{ts'} \quad \forall n \in \{1.......,N_t\}, \forall t \in \{1.......,T-1\}, s \neq s' \in \Lambda_{tn} \qquad (2.15)$$

where Λ_{tn} is the set of scenarios passing through node (t, n)

Most multi-stage SP are computationally intractable because as the number of scenarios increase exponentially as the number of stages increases. This section has provided a brief introduction to SP; the next section describes several applications of SP to problems faced in the power industry.

2.2.2 Recent use of stochastic programming in power systems applications

SP is one of the major tool used by electric power engineers to solve problems when faced with uncertainty. Several authors have applied SP to a wide variety of areas in the power industry, especially in the areas of electricity markets, generation and transmission expansion planning.

Carrion et al. [10] applied SP to energy supply to large consumers through contracts. Carrion et al. [11] also applied SP to vulnerability based transmission expansion planning. The idea is how to optimally re-inforce the transmission network in order to mitigate deliberate attacks. Vulnerability is measured in terms of the expected load shed. Banzo et al. [12] applied SP to planning of offshore wind farms. Gil et al. [13] applied stochastic mixed-integer programming to generation capacity expansion planning under hydro uncertainty.

Konstantelos et al. [14] used SP to solve transmission expansion problem. However, due to the fact that transmission investment are capital intensive and irreversible, other non-transmission options are considered such as phase-shifting transformers, energy storage and demand-side management. These non-transmission options are considered as flexible solution. The problem is formulated as an SP that evaluates both transmission investment and other flexible options under Uncertainty.

SP has been widely applied to unit commitment, Zheng et al. [15] compiled a comprehensive review of different formulation of unit commitment solved by SP. Qadrdan et al. [16] explored the operation of an integrated gas and electricity network in Great Britain. The uncertainty considered was wind power forecast. The problem was solved using both two-stage SP and multi-stage SP. SP was found to reduce operations costs as compared to a deterministic formulation. Tan et al. [17] formulated a two-stage SP that considers the risk level for distribution networks operation with wind power, in this problem the first-stage solution is the wind dispatched while the second-stage considers the difference between dispatched wind power and the actual wind power and also considers the cost of operational risk is also computed.

Marí et al. [18] used applied SP to planning of renewables for a medium-term horizon. A scenario tree was developed using a quasi-Monte Carlo approach considering uncertainties in wind power generation, solar photovoltaic generation and hydro inflows. Munoz et al. [19] applied SP to transmission planning under market and regulatory uncertainty. The two-stage SP model was used. The stochastic solution was compared to deterministic planning based on individual solutions and a heuristic solution that combined results from different deterministic plans. The stochastic solution performed better than the deterministic plans and heuristics plans.

Aasgard et al. [20] applied SP to a market bidding model of hydropower producer participants. The uncertainty modelled is water inflow and spot market prices. The optimization formulation was a stochastic MILP for bid optimization. The stochastic model was compared to a deterministic formulation and performed better in the area of average prices. Romero et al. [21] used a two-stage SP model to solve transmission and generation expansion under seismic risk. The two-stage SP minimizes the expected generation, load shed and repair costs in selected recovery periods

2.3 Robust Optimization

A new approach used for electric power planning under uncertainty is known as robust optimization. Unlike SP there is no need for specific probability distributions for the random variables. This can be seen an advantage of because sometimes this probability distribution are either unknown or difficult to get. When probability distributions for the random variables are exact, this is a very strong assumption. Sometimes when new uncertainty arises, there is no previous information to model the probability distributions. Determining probability distributions wrongly can have dire consequences. One of the drawbacks of robust optimization is that results are overly conservative. There are three major ways in which uncertainty sets are modelled in robust optimization: box, ellipsoidal and polyhedral sets [22]. The conservativeness can be modified by adjusting the uncertainty sets [23].

2.3.1 Formulation

This section describes mathematical formulation of robust optimization. The standard form of linear programming can be written as:

$$\text{Min } c'x \tag{2.16}$$

$$\text{s.t } Ax \geq b \tag{2.17}$$

$$x \in X \tag{2.18}$$

The robust formulation can be written as [24]

$$\text{Min } c'x \tag{2.19}$$

$$\text{s.t } Ax \leq b \quad \forall A \in U \tag{2.20}$$

$$x \in X \tag{2.21}$$

where U is the uncertainty set

2.3.2 Recent use of robust optimization in power system application

Jabr [25] used robust optimization for transmission network planning under uncertainty, uncertain parameters were renewable generation and loads. The range of variation of renewable generation and loads are modelled using box uncertainty sets. A minimum and maximum value was assigned to both uncertain parameters. Chen et al. also used robust optimization for transmission network planning under uncertainty, uncertain parameters were generation and loads. Jabr et al. [26] applied robust optimization to investment of storage facilities on transmission network. This approach called ROSION—"Robust Optimization of Storage Investment On Networks," ensures that system is operated without load or renewable power curtailment. The computational approach used by ROSION is column-and-constraint generation algorithm. The uncertainty modelled are extreme operating conditions that the system could encounter during a planning horizon.

Wu et al. [27] applied robust optimization to wind power look-ahead dispatch. The idea is to economically dispatch conventional generator in the presence of wind power uncertainty. The objective function consists of two costs, the first cost is the cost of conventional generation and the penalty cost for the curtailment of wind power. The output solution consists of the dispatch for conventional generators and an allowable interval for wind generation output, hence reducing the uncertainty associated with the availability of wind generation capacity.

Dehghan et al. [28] applied robust optimization to generation expansion planning. The problem is modelled as mixed-integer linear programming problem. The uncertainty considered were demand and both estimated investment and operations costs. The polyhedral uncertainty set is used to model these uncertainty. Bender's decomposition is applied to solve the MILP Model. Wang et al. [29] applied robust optimization to optimal placement of DG's in a micro grid. The objective function was formulated to minimize the difference of revenues (i.e. payment by Load serving entity (L.S.E) and utility customers) and investment, O&M costs, fuel costs, emission costs. The problem is formulated as two-stage robust optimization problem and the uncertainty considered is DG output and load consumption pattern. The polyhedral uncertainty set is used for both DG output and load consumption uncertainty. The column-and-constraint generation algorithm is used to solve the problem.

Xiong et al. [30] formulated an adjustable robust optimization to solve the unit commitment problem. The problem is modelled as a two-stage robust optimization. The objective is to minimize costs of generation and load shedding costs under the worst case scenario considering the uncertainty set .The uncertainty considered was generator unavailability and demand variability. The polyhedral uncertainty set is used for both generator unavailability and demand variability.

Lee et al. [31] used robust optimization to solve unit commitment problem while modelling transmission line constraints. The problem is a two-stage robust optimization. The two uncertainty considered were load and wind power generation. The polyhedral uncertainty set is used for both load and wind power generation uncertainty. The objective function was formulated to minimize was the sum of cost of generation and the worst-case dispatch cost. The column-and-constraint generation algorithm is used to solve the problem. The uncertainty considered were load and wind power generation. Moreira et al. [32] applied robust optimization to security-constrained transmission expansion planning. The authors apply the N-k security criterion. The problem is modelled as a tri-level programming approach.

Multiple researchers have applied robust optimization to many applications in power systems, however despite being a well-known approach,it tends to provide overly-conservative results.

2.4 Real Options

Real options analysis was introduced by Stewart Myers in 1977 [33]. A real option is the right, but not the obligation to undertake an investment decision: usually the option is to delay, expand, abandon or reduce a capital investment. The real-options approach applies financial options theory to real investments (i.e. the concept of option pricing techniques for financial securities is applied to real investment) and focuses on managerial flexibility. Managerial flexibility refers to the flexibility a firm has in terms of scaling and timing of an investment decision as market conditions change. The following can be described as types of real options" [34].

1.) Option to Abandon
2.) Option to wait and see
3.) Option to delay
4.) Option to expand
5.) Option to contract
6.) Option to choose
7.) Option to switch resources

Recent use of real options in power system application

Jo Min et al. [35] applied the concept of real options to study the impact of entry and exit of investment for renewables power producers. In this study, a single renewable site is used. The

O&M costs of wind is modelled as Geometric Brownian Motion Distribution (GBM). This study examines the entry and exit options available to renewable site decision makers.

Blanco et al. [36] used real option for the valuation of flexible ac transmission systems (FACTS) using the Least Square Monte Carlo Method. The authors try to capture the value of deferring investment in transmission lines which have large capital costs by investing in (FACTS) devices. The authors view (FACTS) as a tool that adds flexibility to the transmission expansion planning.

Ramanathan et al. [37] applies real options to analyze transmission investments in a deregulated environment. In this formulation framework, options such as expanding existing transmission lines, delaying transmission investment, and compound options are analyzed for transmission investment. Binomial trees analysis with embedded Monte-Carlo simulation is used for real options analysis.

Hedman et al. [38] give an overview of the application of real options to transmission expansion planning. Approaches in real options that can be used to assess the value of transmission lines investment are described. The authors describe how managerial flexibility can more accurately assess the value of transmission planning as opposed to NPV.

The issue with real options is that it is limited in its scope because the most important parameter in real options is volatility, and in a problem where the issue of volatility is not very important, its use becomes limited.

2.5 Adaptation

The main idea behind adaptation is to design a common transmission network, a core that can be adapted to future scenarios. This concept was developed by a former PhD student at Iowa State University [39]; however it was applied only to generation expansion planning. This research plans to use the adaptation to design flexible R2B solutions, transmission and co-optimization planning under uncertainty. The core transmission network is designed so that it can adapt to future scenarios at minimum cost. What makes the adaptation unique is that rather than selecting a flexible plan, it designs a flexible plan. In this approach, a system is flexible if it can be adapted cost-efficiently to the conditions of any other scenario. Figure 4 below depicts the idea of the adaptation, which is to choose a core design ("Core Des" in the figure) because it minimizes the core cost plus the cost of adapting to the various possible futures.

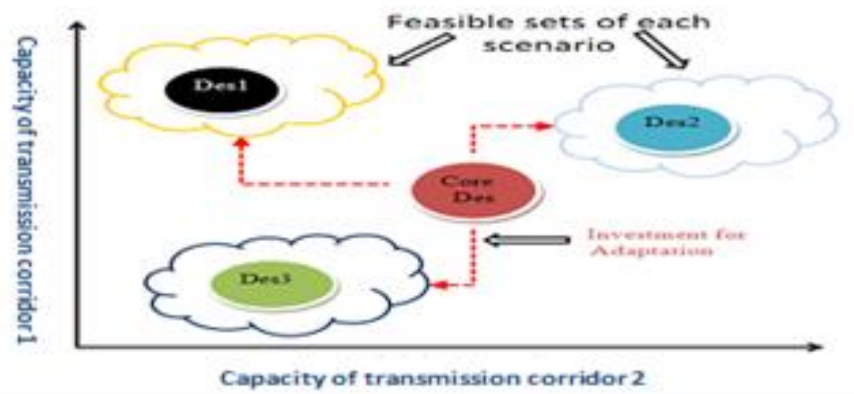

Figure 4: Conceptualization of adaptation

A general form of adaptation is to designate the investments associated with a "core" plan by the vector x^f. These investments describe a decision to build infrastructure, independent of what future occurs. We then identify possible futures that may occur, denoted by i=1,…,N, and we describe these futures by constraints $g_i(x^f+\Delta x_i)\leq b_i$, where the Δx_i are an additional set of decision variables that represent the change in investments, relative to the core investments, if scenario i occurs. There is a cost to the Δxi which represents the cost of adapting the core to scenario i. The goal is, then, to minimize the total cost, that is, to minimize the cost of the core investments plus the adaptation cost. Of course, we may prefer to avoid incurring core costs and mainly rely on adaptation; alternatively, we may prefer to avoid adaptation costs and mainly relay on core investments. These two extremes, and various other preferences in between, can be achieved by weighting the adaptation costs appropriately. We model the corresponding scalar weight as β. This leads to the general expression of adaptation, as follows:

$$\text{Min} \quad CoreCosts(\underline{x}^f) + \beta\left[\sum_i AdaptationCosts(\Delta\underline{x}_i)\right] \tag{2.22}$$

s.t

$$Constraints\ \ for\ \ scenario\ \ i=1,.......N:\ g_i(\mathrm{x}_i+\Delta\,\mathrm{x}_i)\leq \mathrm{b}_i \tag{2.23}$$

$\underline{x}^f$ Core investments, to be used by all scenarios i

$\Delta\underline{\mathrm{x}}_i$ Additional investments needed to adapt to scenarios i

Mejia and McCalley [40] proposed adaptation for generation expansion planning. This dissertation extends the idea of adaptation to transmission planning and co-optimization of transmission and generation expansion.

CHAPTER 3. ADAPTATION VS STOCHASTIC PROGRAMMING

The purpose of this chapter is to clarify the differences between the well-known approach for solving optimization problems under uncertainty known as stochastic programming (SP) with adaptation. As much as there are differences in these approaches, there are also similarities.

3.1 Formulations

This section describes the formulations for SP and for adaptation. We do not provide comprehensive formulations of each but rather provide formulations that enable identification of the basic differences between the approaches.

3.1.1 Stochastic programming

A general form for the SP formulation is given as follows:

$$Minimize\ I_c Cap_1^c + \sum_{t,w} \Pr^w (I_{t,w} \Delta Cap_t^w + OC_t^w) \qquad (3.1)$$

Subject to

$$Cap_t^w = Cap_{t-1}^w + \Delta Cap_t^w \quad \forall, t, w \qquad (3.2)$$

$$Cap_1^w = Cap_1^c \qquad (3.3)$$

Plus non-anticipative constraints (see section 2.2)

Plus operational constraints for each scenario w.

where:

I_c is the investment costs of the initial investment

Cap_1^c is the initial capacity investment

Pr^w is the probability of occurrence of scenario w

OC_t^w is the operation costs of scenario w at time t

ΔCap_t^w is the capacity needed to adapt to scenario w at time t

w designates the scenario

Cap_t^w is the capacity at time t in scenario w

Cap_{t-1}^w is the capacity at time t-1 in scenario w

Operational constraints include the maximum power a generator is allowed to dispatch in an operating condition. These constraints are not analytically expressed here because they are similar for both SP and adaptation formulations, and so their presence obscures the main differences between the two approaches without providing additional insight.

The first step in SP is to build a scenario tree and then assign probabilities to future outcomes and finally optimize over all possibilities. Equation (3.2) describes capacity update for each scenario, where the update at time t is summed with the previous capacity update. Equation (3.3) depicts that the initial investment is used for all scenarios. Operations costs is the cost of generation dispatch for the planning horizon.

3.1.2 Adaptation

A general form for the adaptation formulation is given as follows:

$$Minimize\ I_t Cap_t^{add} + \sum_{t,w} (\Pr^w OC_t^w) + \beta \sum_{t,w} \Pr^w I_t^w \Delta Cap_t^w \tag{3.4}$$

Subject to

$$Cap_t^c = Cap_{t-1}^c + Cap_t^{add} \quad \forall t \tag{3.5}$$

Core update equation, from t-1 to t

$$Cap_t^w = Cap_t^c + \Delta Cap_t^w \;\; \forall t, w \tag{3.6}$$

Adaptation equation, from core to future w.

$$Cap_0^c = 0 \tag{3.7}$$

where:

I_t is investment costs at time t

$Cap^0{}_c$ is the initial capacity of the core-trajectory

$Cap^t{}_c$ is the core-capacity trajectory at time t

$Cap_{t-1}{}^c$ is the core-capacity trajectory at time t-1

$Cap_t{}^{add}$ is the capacity added to the core-trajectory at time t

w designates the scenario

$Cap^t w$ is the capacity at time t in scenario w

Pr_w is the probability of occurrence of scenario w

$OC^t{}_w$ is the operation costs of scenario w at time t

$\Delta Cap_t{}^w$ is the capacity needed to adapt to scenario w at time t

$I_t{}^w$ is the investment costs at time t in scenario w

β is a trade-off parameter

In adaptation, the objective function uses parameter β to multiply the costs of adapting to scenarios. The equation (3.5) can be seen as the core-update equation where, at each time/stage when decisions are made, the core-trajectory is updated. The equation (3.6) is the adaptation equation, where the core-trajectory is adapted to different futures at time t.

3.2 Comparison

This section discusses the formulation differences, differences in the treatment of uncertainty, conceptual differences and their complexity analysis.

3.2.1 Formulation differences

In adaptation, ΔCap_{tw} depends only on the relation between the core at time t and the feasible set for scenario w at time t, and is independent of ΔCap_{t-1w}. This is not the case in SP where future decisions are conditional on previous decisions made in a particular scenario. In SP the decision ΔCap_{tw} is conditional on the decision ΔCap_{t-1w} and all previous decisions before ΔCap_{t-1w}.In adaptation, the core-trajectory is updated throughout the planning horizon and common to all scenarios, while in SP only initial plan is common to all scenarios.

3.2.2 Differences in treatment of uncertainty

In SP, as the number of stages increases the number of scenarios increases, while in the adaptation, as the number of stages increases, the number of scenarios does not increase. SP is a technique for making sequential decisions under uncertainty. The fact that SP scenarios increase as the number of stages increases, can make SP computationally too expensive. The goal of SP is to maximize the expected return modelled as a probabilistic objective function, while the goal of adaptation is to minimize the cost of the core investments plus the adaptation cost. SP is effectively visualized via a scenario tree, as indicated in Figure 5. Denoting each red circle as a state and each vertically aligned group of red circles as a stage (in time), we observe that at each state in each stage in the scenario tree there exists a transition probability to move from one state to another in the next stage.

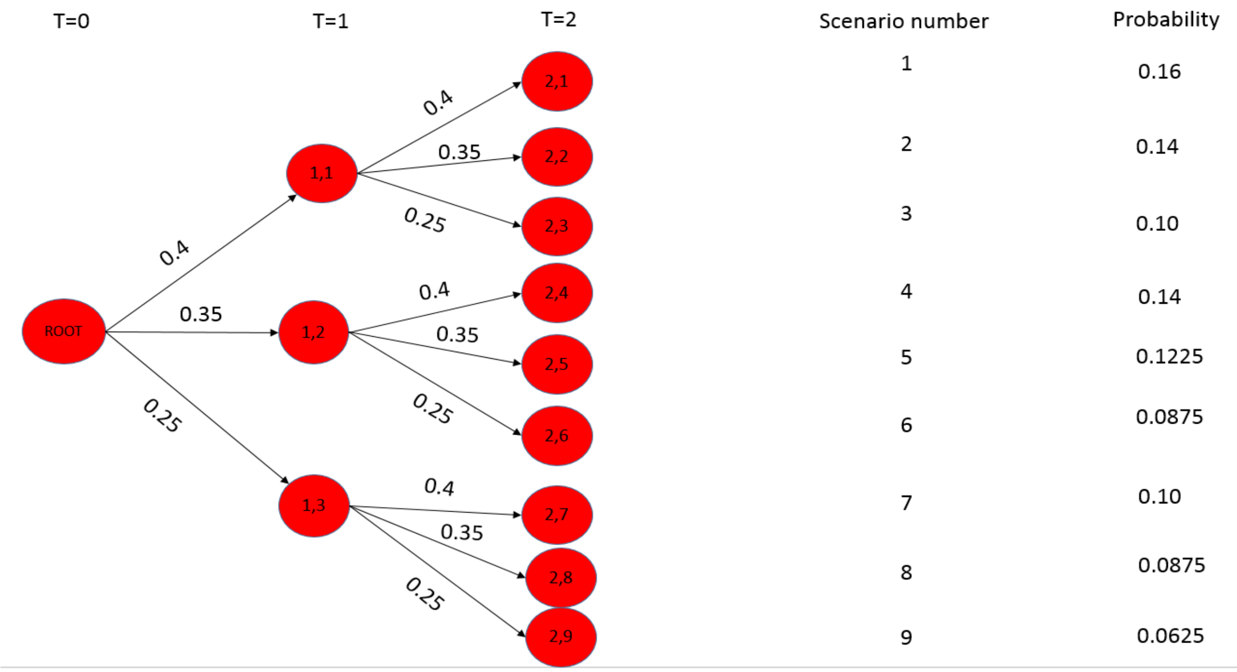

Figure 5: Scenario tree in stochastic programming

SP develops a strategy for all the paths in a scenario tree. For example in Fig. 5 there is a solution strategy for all 9 paths; in addition, SP requires enforcement of non-anticipatively (i.e. all paths have the same solution until they split). In contrast, adaptation tries to find a trajectory which is "close" to each of the scenario feasibility sets.

In adaptation we refer to uncertainties using what we call local and global uncertainties [40].

a).Global - uncertainties for which different values produce dramatically different results: emissions policies, large demand shifts, coal or nuclear unavailability, extremes in fuel prices, extended drought, dramatic change in technology investment costs. Within each global uncertainty we have multiple local uncertainties. The uncertainty is represented as a specified set of trajectories through the time intervals, one for each defined "future," where each trajectory represents a set of realizations on global uncertainties at each stage or time interval.

b) Local - range of values a parameter may take under a global realization: variation in load growth, investment costs or fuel prices, e.g, demand growth that is “high” (e.g., 5%) vs. demand growth that is “low” (e.g., 0.5%).   Local uncertainties refer to those for which the uncertain parameter varies about a central value, e.g., expected demand growth of 1.5% with 3-sigma deviation of 0.5.

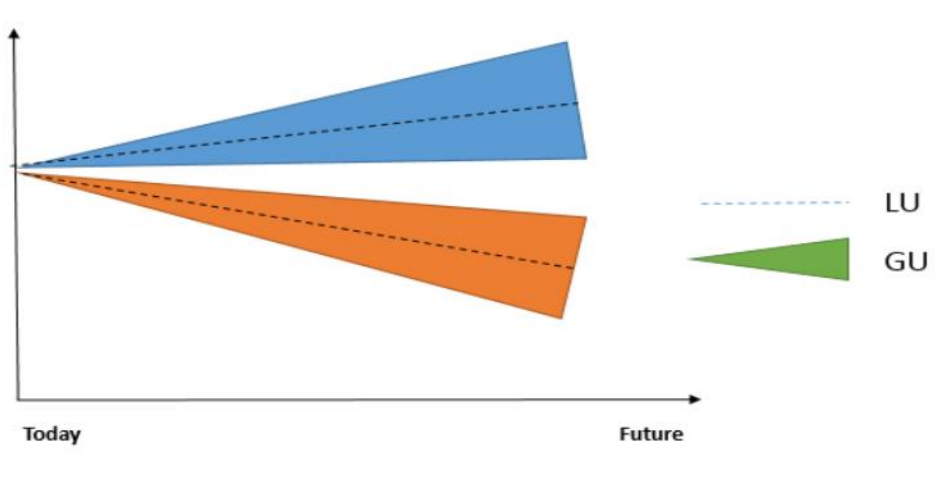

Figure 6: Global and local Uncertainties

3.2.3 Conceptual differences

This section discusses conceptual differences between SP and adaptation. A pictorial comparison of both approaches in a multi-stage planning horizon is shown in Figs. 7 and 8.

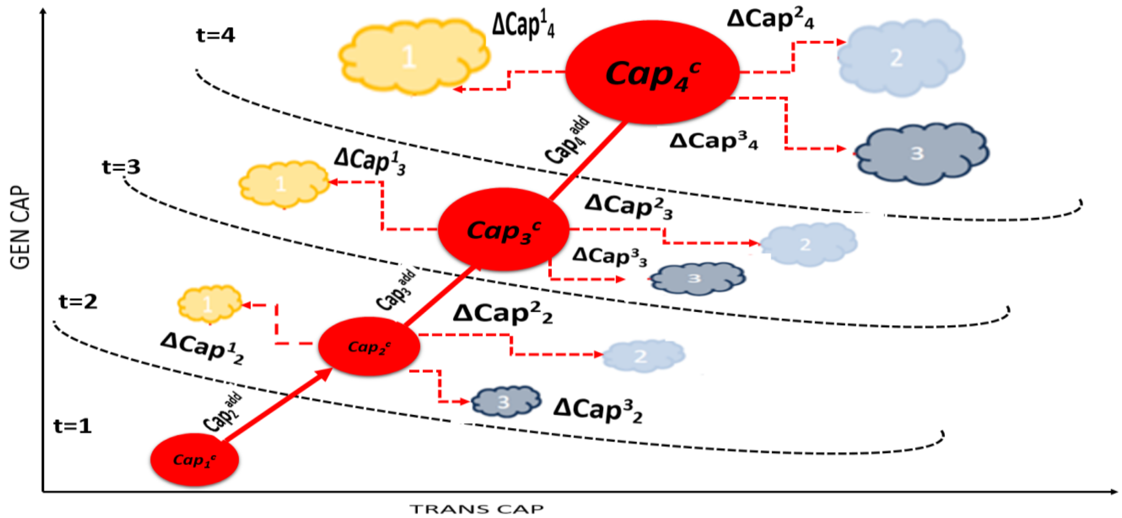

Figure 7: Description of adaptation

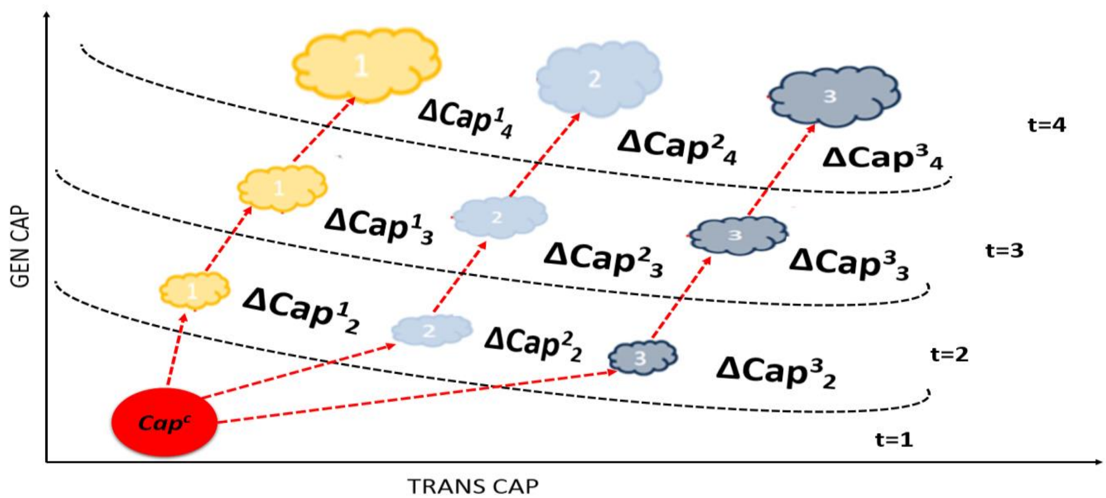

Figure 8: Description of stochastic programming

In adaptation illustrated in Fig. 7, we find through Cap^c, a

"central" investment trajectory

that in each period, is "close" to each of the scenario feasibility sets, so that the infrastructure design will be able to effectively transition to a "good" solution if, during a given period, one of the particular scenarios occurs. In contrast, with SP, we find through Cap^c, an

initial investment

to most effectively facilitate the investment needs of future periods.

For both of the above diagrams, the **red** circle represents the core transmission/generation investment that is invested, accounting for all scenarios. There are three time intervals t=1, 2, 3; and three futures through those time intervals (in each time interval, the three clouds represent the futures at that time). The dashed arrows represent the added transmission/generation investment necessary to adapt to each of the three futures at each time interval.

3.2.4 Complexity Analysis

Computational complexity "measures how much work is required to solve different problems" [41]. The purpose of this section is to compare and contrast adaptation and SP based on factors that can increase the complexity of each of these approaches. Both of these problems are mixed integer linear programs (MILP).

A MILP is known to be NP-hard, meaning it is at least as hard as any NP problem. NP (nondeterministic polynomial time) are a "set of problems for which a solution can be efficiently verified" [42].

For NP-hard problems, there is generally no single factor that determines problem complexity. However, for a MILP problem, it is known that complexity is highly influenced by the number of variables, particularly the integer variables, and by the number of constraints. As a result, in order to compare the computational complexity of adaptation to SP, we will, in this dissertation, compare their respective number of variables and number of constraints.

In our effort to compare complexity of SP with that of adaptation, we will make the following assumptions.

- The SP is a two-stage approximation. This is relevant because in a two-stage approximation, after the first stage, uncertainty is revealed for all future stages, and the uncertainty structure has a fan-like structure. This is unlike the multi-stage SP where the uncertainty structure is tree-like.

- The two approaches use the same uncertainty set.
- The comparison is based on the transmission planning formulation (and not the co-optimization formulation) for adaptation and for SP with two-stage approximation The detailed transmission expansion formulation for adaptation can be found in Chapter 4.

ILLUSTRATION

This section illustrates the concept of SP and adaptation on a simple 3-system. The planning horizon is 3 years and 2 futures/scenarios are considered. The constraints required to solve this problem is written for both SP and adaptation. The base load is assumed to be 500MW.

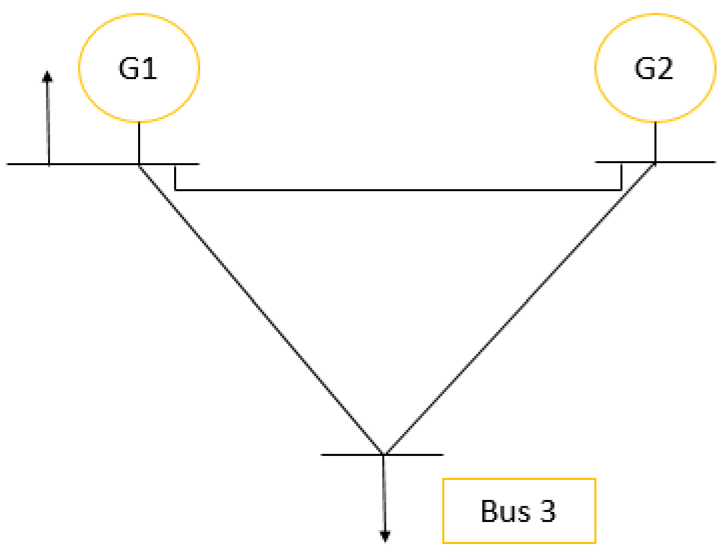

Figure 9: 3-bus system

Table 1: Branch data for existing lines

Existing lines	Capacity	Reactance
1-2	200MW	0.15
1-3	200MW	0.20
2-3	200MW	0.25

Table 2: Branch data for candidate lines

Existing lines	Capacity	Reactance
1-2	300MW	0.15
1-3	300MW	0.20

In each year we assume there are 2 operating conditions.

1) 50% of peak load

2) Peak load

Two futures/scenarios are assumed

1) Low load growth

2) High load growth

Table 3: load growth for the two futures considered

Scenario	T=1	T=2	T=3
Low load growth	505MW	510.05MW	515.15MW
High load growth	510MW	520.20MW	530.60MW

The types of constraints required for transmission expansion planning are divided into 5 types.

1) Power – demand balance constraints
2) Line-flow constraints for existing lines
3) Line-flow constraints for candidate lines
4) Capacity limit constraints for candidate lines
5) Special constraints unique to both approaches

The first two types of constraints are the same for both approaches, so they are written once, while the last three types of constraints are different, so they are written both approaches.

For the nomenclature, a sample of variables from each type of variable is defined and the rest follow the same pattern.

Nomenclature

$f^0_{1,2,1,1,1}$ is the line flow for existing line that goes from bus 1 to bus 2 in operating condition 1 in year 1 and in scenario 1

$f^c_{1,2,1,1,1}$ is the line flow for candidate line that goes from bus 1 to bus 2 in operating condition 1 in year 1 and in scenario 1

$P_{1,1,1,1}$ is the dispatched power from generator 1 in operating condition 1 in year 1 and in scenario 1

$\theta_{2,1,1,1}$ is the angle at bus 2 in operating condition 1 in year 1 and in scenario 1

$X_{1,0}$ is the binary variable for transmission line candidate 1 at t=0

$X_{1,1}$, $X_{1,2}$ and $X_{1,3}$ is the binary variable for transmission line for candidate 1 at t=1,t=2,t=3 (variable available only in adaptation formulation for core-trajectory)

$X_{1,1,1}$ is the binary variable for transmission line for candidate 1 at time 1 in scenario 1

Parameters

We first define parameters used to compare the two approaches.

W, the number of scenarios;

B, the number of transmission candidates in binary variables;

TB_{sp}, the total number of binary variables in an SP formulation;

TB_a, the total number of binary variables in an adaptation formulation;

TC_{sp}, the total number of continuous variables in an SP formulation;

TC_a, the total number of continuous variables in an adaptation formulation;

C, the number of operating conditions in a year;

E, the number of existing lines;

NB, the number of buses.

T is the number of years in the planning horizon

1) **Power – demand balance constraints**

Power demand balance equations for the 1st operating condition and for year 1 in scenario 1:

$$f^{0}_{1,2,1,1,1} + f^{c}_{1,2,1,1,1} + f^{0}_{1,3,1,1,1} + P_{1,1,1,1} - 101 = 0 \tag{3.8}$$

$$-f^{0}_{1,3,1,1,1} + f^{0}_{2,3,1,1,1} + + f^{c}_{2,3,1,1,1} + P_{2,1,1,1} = 0 \tag{3.9}$$

$$-f^{0}_{1,2,1,1,1} - f^{c}_{1,2,1,1,1} - f^{0}_{2,3,1,1,1} - f^{c}_{2,3,1,1,1} - 151.5 = 0 \tag{3.10}$$

Power demand balance equations for the 2nd operating condition and for year 1 in scenario 1:

$$f^{0}_{1,2,2,1,1} + f^{c}_{1,2,2,1,1} + f^{0}_{1,3,2,1,1} + P_{1,2,1,1} - 202 = 0 \tag{3.11}$$

$$-f^{0}_{1,3,2,1,1} + f^{0}_{2,3,2,1,1} + + f^{c}_{2,3,2,1,1} + P_{2,2,1,1} = 0 \tag{3.12}$$

$$-f^{0}_{1,2,2,1,1} - f^{c}_{1,2,2,1,1} - f^{0}_{2,3,2,1,1} - f^{c}_{2,3,2,1,1} - 303 = 0 \tag{3.13}$$

Power demand balance equations for the 1st operating condition and for year 2 in scenario 1

$$f^{0}_{1,2,1,2,1} + f^{c}_{1,2,1,2,1} + f^{0}_{1,3,1,2,1} + P_{1,1,2,1} - 102.01 = 0 \tag{3.14}$$

$$-f^{0}_{1,3,1,2,1} + f^{0}_{2,3,1,2,1} + + f^{c}_{2,3,1,2,1} + P_{2,1,2,1} = 0 \tag{3.15}$$

$$-f^{0}_{1,2,1,2,1} - f^{c}_{1,2,1,2,1} - f^{0}_{2,3,1,2,1} - f^{c}_{2,3,1,2,1} - 153.015 = 0 \tag{3.16}$$

Power demand balance equations for the 2nd operating condition and for year 2 in scenario 1:

$$f^{0}_{1,2,2,2,1} + f^{c}_{1,2,2,2,1} + f^{0}_{1,3,2,2,1} + P_{1,2,2,1} - 204.02 = 0 \tag{3.17}$$

$$-f^{0}_{1,3,2,2,1}+f^{0}_{2,3,2,2,1}++f^{c}_{2,3,2,2,1}+P_{2,2,2,1}=0 \tag{3.18}$$

$$-f^{0}_{1,2,2,2,1}-f^{c}_{1,2,2,2,1}-f^{0}_{2,3,2,2,1}-f^{c}_{2,3,2,2,1}-306.03=0 \tag{3.19}$$

Power demand balance equations for the 1st operating condition and for year 3 in scenario 1:

$$f^{0}_{1,2,1,3,1}+f^{c}_{1,2,1,3,1}+f^{0}_{1,3,1,3,1}+P_{1,1,3,1}-103.03=0 \tag{3.20}$$

$$-f^{0}_{1,3,1,3,1}+f^{0}_{2,3,1,3,1}++f^{c}_{2,3,1,3,1}+P_{2,1,3,1}=0 \tag{3.21}$$

$$-f^{0}_{1,2,1,3,1}-f^{c}_{1,2,1,3,1}-f^{0}_{2,3,1,3,1}-f^{c}_{2,3,1,3,1}-154.545=0 \tag{3.22}$$

Power demand balance equations for the 2nd operating condition and for year 3 in scenario 1:

$$f^{0}_{1,2,2,3,1}+f^{c}_{1,2,2,3,1}+f^{0}_{1,3,2,3,1}+P_{1,2,3,1}-206.06=0 \tag{3.23}$$

$$-f^{0}_{1,3,2,3,1}+f^{0}_{2,3,2,3,1}++f^{c}_{2,3,2,3,1}+P_{2,2,3,1}=0 \tag{3.24}$$

$$-f^{0}_{1,2,2,3,1}-f^{c}_{1,2,2,3,1}-f^{0}_{2,3,2,3,1}-f^{c}_{2,3,2,3,1}-309.09=0 \tag{3.25}$$

Power demand balance equations for the 1st operating condition and for year 1 in scenario 2:

$$f^{0}_{1,2,1,1,2}+f^{c}_{1,2,1,1,2}+f^{0}_{1,3,1,1,2}+P_{1,1,1,2}-102=0 \tag{3.26}$$

$$-f^{0}_{1,3,1,1,2}+f^{0}_{2,3,1,1,2}++f^{c}_{2,3,1,1,2}+P_{2,1,1,2}=0 \tag{3.27}$$

$$-f^{0}_{1,2,1,1,2}-f^{c}_{1,2,1,1,2}-f^{0}_{2,3,1,1,2}-f^{c}_{2,3,1,1,2}-153=0 \tag{3.28}$$

Power demand balance equations for the 2nd operating condition and for year 1 in scenario 2:

$$f^{0}_{1,2,2,1,2}+f^{c}_{1,2,2,1,2}+f^{0}_{1,3,2,1,2}+P_{1,2,1,2}-204=0 \tag{3.29}$$

$$-f^{0}_{1,3,2,1,2}+f^{0}_{2,3,2,1,2}++f^{c}_{2,3,2,1,2}+P_{2,2,1,2}=0 \tag{3.30}$$

$$-f^{0}_{1,2,2,1,2}-f^{c}_{1,2,2,1,2}-f^{0}_{2,3,2,1,2}-f^{c}_{2,3,2,1,2}-306=0 \tag{3.31}$$

Power demand balance equations for the 1st operating condition and for year 2 in scenario 2:

$$f^{0}_{1,2,1,2,2} + f^{c}_{1,2,1,2,2} + f^{0}_{1,3,1,2,2} + P_{1,1,2,2} - 104.04 = 0 \quad (3.32)$$

$$-f^{0}_{1,3,1,2,2} + f^{0}_{2,3,1,2,2} + + f^{c}_{2,3,1,2,2} + P_{2,1,2,2} = 0 \quad (3.33)$$

$$-f^{0}_{1,2,1,2,2} - f^{c}_{1,2,1,2,2} - f^{0}_{2,3,1,2,2} - f^{c}_{2,3,1,2,2} - 156.06 = 0 \quad (3.34)$$

Power demand balance equations for the 2nd operating condition and for year 2 in scenario 2:

$$f^{0}_{1,2,2,2,2} + f^{c}_{1,2,2,2,2} + f^{0}_{1,3,2,2,2} + P_{1,2,2,2} - 208.08 = 0 \quad (3.35)$$

$$-f^{0}_{1,3,2,2,2} + f^{0}_{2,3,2,2,2} + + f^{c}_{2,3,2,2,2} + P_{2,2,2,2} = 0 \quad (3.36)$$

$$-f^{0}_{1,2,2,2,2} - f^{c}_{1,2,2,2,2} - f^{0}_{2,3,2,2,2} - f^{c}_{2,3,2,2,2} - 312.12 = 0 \quad (3.37)$$

Power demand balance equations for the 1st operating condition and for year 3 in scenario 2:

$$f^{0}_{1,2,1,3,2} + f^{c}_{1,2,1,3,2} + f^{0}_{1,3,1,3,2} + P_{1,1,3,2} - 106.12 = 0 \quad (3.38)$$

$$-f^{0}_{1,3,1,3,2} + f^{0}_{2,3,1,3,2} + + f^{c}_{2,3,1,3,2} + P_{2,1,3,2} = 0 \quad (3.39)$$

$$-f^{0}_{1,2,1,3,2} - f^{c}_{1,2,1,3,2} - f^{0}_{2,3,1,3,2} - f^{c}_{2,3,1,3,2} - 159.18 = 0$$

Power demand balance equations for the 2nd operating condition and for year 3 in scenario 2:

$$f^{0}_{1,2,2,3,2} + f^{c}_{1,2,2,3,2} + f^{0}_{1,3,2,3,2} + P_{1,2,3,2} - 212.24 = 0 \quad (3.41)$$

$$-f^{0}_{1,3,2,3,2} + f^{0}_{2,3,2,3,2} + + f^{c}_{2,3,2,3,2} + P_{2,2,3,2} = 0 \quad (3.42)$$

$$-f^{0}_{1,2,2,3,2} - f^{c}_{1,2,2,3,2} - f^{0}_{2,3,2,3,2} - f^{c}_{2,3,2,3,2} - 318.36 = 0 \quad (3.43)$$

The first set of constraints are power demand balance equations for the whole planning horizon.

The number of these equality constraints can be computed using the formula.

=NB*C*T*W

=3*2*3*2

=36

2.) Line-flow constraints for existing lines

Line-flow constraints for existing lines in the 1st operating condition, for year 1 in scenario 1:

$$f^{0}_{1,2,1,1,1} - 6.67(\theta_{1,1,1,1} - \theta_{2,1,1,1}) = 0 \tag{3.44}$$

$$f^{0}_{1,3,1,1,1} - 5(\theta_{1,1,1,1} - \theta_{3,1,1,1}) = 0 \tag{3.45}$$

$$f^{0}_{2,3,1,1,1} - 4(\theta_{2,1,1,1} - \theta_{3,1,1,1}) = 0 \tag{3.46}$$

The line-flow constraints for existing lines in the 2nd operating condition and for year 1 in scenario 1:

$$f^{0}_{1,2,2,1,1} - 6.67(\theta_{1,2,1,1} - \theta_{2,2,1,1}) = 0 \tag{3.47}$$

$$f^{0}_{1,3,2,1,1} - 5(\theta_{1,2,1,1} - \theta_{3,2,1,1}) = 0 \tag{3.48}$$

$$f^{0}_{2,3,2,1,1} - 4(\theta_{2,2,1,1} - \theta_{3,2,1,1}) = 0 \tag{3.49}$$

The line-flow constraints for existing lines for the 1st operating condition and for year 2 in scenario 1:

$$f^{0}_{1,2,1,2,1} - 6.67(\theta_{1,1,2,1} - \theta_{2,1,2,1}) = 0 \tag{3.50}$$

$$f^{0}_{1,3,1,2,1} - 5(\theta_{1,1,2,1} - \theta_{3,1,2,1}) = 0 \tag{3.51}$$

$$f^{0}_{2,3,1,2,1} - 4(\theta_{2,1,2,1} - \theta_{3,1,2,1}) = 0 \tag{3.52}$$

The line-flow constraints for existing lines for the 2nd operating condition and for year 2 in scenario 1:

$$f^{0}_{1,2,2,2,1} - 6.67(\theta_{1,2,2,1} - \theta_{2,2,2,1}) = 0 \tag{3.53}$$

$$f^{0}_{1,3,2,2,1} - 5(\theta_{1,2,2,1} - \theta_{3,2,2,1}) = 0 \tag{3.54}$$

$$f^{0}_{2,3,2,2,1} - 4(\theta_{2,2,2,1} - \theta_{3,2,2,1}) = 0 \tag{3.55}$$

The line-flow constraints for existing lines for the 1st operating condition and for year 3 in scenario 1:

$$f^{0}_{1,2,1,3,1} - 6.67(\theta_{1,1,3,1} - \theta_{2,1,3,1}) = 0 \tag{3.56}$$

$$f^{0}_{1,3,1,3,1} - 5(\theta_{1,1,3,1} - \theta_{3,1,3,1}) = 0 \tag{3.57}$$

$$f^{0}_{2,3,1,3,1} - 4(\theta_{2,1,3,1} - \theta_{3,1,3,1}) = 0 \tag{3.58}$$

The line-flow constraints for existing lines for the 2nd operating condition and for year 3 in scenario 1:

$$f^{0}_{1,2,2,3,1} - 6.67(\theta_{1,2,3,1} - \theta_{2,2,3,1}) = 0 \tag{3.59}$$

$$f^{0}_{1,3,2,3,1} - 5(\theta_{1,2,3,1} - \theta_{3,2,3,1}) = 0 \tag{3.60}$$

$$f^{0}_{2,3,2,3,1} - 4(\theta_{2,2,3,1} - \theta_{3,2,3,1}) = 0 \tag{3.61}$$

The line-flow constraints for existing lines in the 1st operating condition and for year 1 in scenario 2:

$$f^{0}_{1,2,1,1,2} - 6.67(\theta_{1,1,1,2} - \theta_{2,1,1,2}) = 0 \tag{3.62}$$

$$f^{0}_{1,3,1,1,2} - 5(\theta_{1,1,1,2} - \theta_{3,1,1,2}) = 0 \tag{3.63}$$

$$f^{0}_{2,3,1,1,2} - 4(\theta_{2,1,1,2} - \theta_{3,1,1,2}) = 0 \tag{3.64}$$

The line-flow constraints for existing lines in the 2nd operating condition and for year 1 in scenario 2:

$$f^{0}_{1,2,2,1,2} - 6.67(\theta_{1,2,1,2} - \theta_{2,2,1,2}) = 0 \tag{3.65}$$

$$f^{0}_{1,3,2,1,2} - 5(\theta_{1,2,1,2} - \theta_{3,2,1,2}) = 0 \tag{3.66}$$

$$f^{0}_{2,3,2,1,2} - 4(\theta_{2,2,1,2} - \theta_{3,2,1,2}) = 0 \tag{3.67}$$

The line-flow constraints for existing lines for the 1st operating condition and for year 2 in scenario 2:

$$f^{0}_{1,2,1,2,2} - 6.67(\theta_{1,1,2,2} - \theta_{2,1,2,2}) = 0 \tag{3.68}$$

$$f^{0}_{1,3,1,2,2} - 5(\theta_{1,1,2,2} - \theta_{3,1,2,2}) = 0 \tag{3.69}$$

$$f^{0}_{2,3,1,2,2} - 4(\theta_{2,1,2,2} - \theta_{3,1,2,2}) = 0 \tag{3.70}$$

The line-flow constraints for existing lines for the 2nd operating condition and for year 2 in scenario 2:

$$f^{0}_{1,2,2,2,2} - 6.67(\theta_{1,2,2,2} - \theta_{2,2,2,2}) = 0 \tag{3.71}$$

$$f^{0}_{1,3,2,2,2} - 5(\theta_{1,2,2,2} - \theta_{3,2,2,2}) = 0 \tag{3.72}$$

$$f^{0}_{2,3,2,2,2} - 4(\theta_{2,2,2,2} - \theta_{3,2,2,2}) = 0 \tag{3.73}$$

The line-flow constraints for existing lines for the 1st operating condition and for year 3 in scenario 2:

$$f^{0}_{1,2,1,3,2} - 6.67(\theta_{1,1,3,2} - \theta_{2,1,3,2}) = 0 \tag{3.74}$$

$$f^{0}_{1,3,1,3,2} - 5(\theta_{1,1,3,2} - \theta_{3,1,3,2}) = 0 \tag{3.75}$$

$$f^{0}_{2,3,1,3,2} - 4(\theta_{2,1,3,2} - \theta_{3,1,3,2}) = 0 \tag{3.76}$$

The line-flow constraints for existing lines for the 2nd operating condition and for year 3 in scenario 2:

$$f^{0}_{1,2,2,3,2} - 6.67(\theta_{1,2,3,2} - \theta_{2,2,3,2}) = 0 \tag{3.77}$$

$$f^{0}_{1,3,2,3,2} - 5(\theta_{1,2,3,2} - \theta_{3,2,3,2}) = 0 \tag{3.78}$$

$$f^{0}_{2,3,2,3,2} - 4(\theta_{2,2,3,2} - \theta_{3,2,3,2}) = 0 \tag{3.79}$$

The second set of constraints are line-flow constraints for existing line for the whole planning horizon

The number of these equality constraints can be computed using the formula

=E*C*T*W

=3*2*3*2

=36

3) Line-flow constraints for candidate lines

The line-flow constraints for candidate lines in the 1st operating condition and for year 1 in scenario 1:

<u>**SP**</u>

$$\left| f^{c}_{1,2,1,1,1} - 10(\theta_{1,1,1,1} - \theta_{2,1,1,1}) \right| \le M(1 - X_{1,0} - X_{1,1,1}) \tag{3.80}$$

$$\left| f^{c}_{2,3,1,1,1} - 6.67(\theta_{2,1,1,1} - \theta_{3,1,1,1}) \right| \le M(1 - X_{2,0} - X_{2,1,1}) \tag{3.81}$$

<u>**Adaptation**</u>

$$\left| f^{c}_{1,2,1,1,1} - 10(\theta_{1,1,1,1} - \theta_{2,1,1,1}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,1,1}) \quad (3.82)$$

$$\left| f^{c}_{2,3,1,1,1} - 6.67(\theta_{2,1,1,1} - \theta_{3,1,1,1}) \right| \le M(1 - X_{2,0} - X_{2,1} - X_{2,1,1}) \quad (3.83)$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 2 in scenario 1:

SP

$$\left| f^{c}_{1,2,2,1,1} - 10(\theta_{1,2,1,1} - \theta_{2,2,1,1}) \right| \le M(1 - X_{1,0} - X_{1,1,1}) \quad (3.84)$$

$$\left| f^{c}_{2,3,2,1,1} - 10(\theta_{2,2,1,1} - \theta_{3,2,1,1}) \right| \le M(1 - X_{2,0} - X_{2,1,1}) \quad (3.85)$$

Adaptation

$$\left| f^{c}_{1,2,2,1,1} - 10(\theta_{1,2,1,1} - \theta_{2,2,1,1}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,1,1}) \quad (3.86)$$

$$\left| f^{c}_{2,3,2,1,1} - 10(\theta_{2,2,1,1} - \theta_{3,2,1,1}) \right| \le M(1 - X_{2,0} - X_{2,1} - X_{2,1,1}) \quad (3.87)$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 2 in scenario 1:

SP

$$\left| f^{c}_{1,2,1,2,1} - 10(\theta_{1,1,2,1} - \theta_{2,1,2,1}) \right| \le M(1 - X_{1,0} - X_{1,1,1} - X_{1,2,1}) \quad (3.88)$$

$$\left| f^{c}_{2,3,1,2,1} - 6.67(\theta_{2,1,2,1} - \theta_{3,1,2,1}) \right| \le M(1 - X_{2,0} - X_{2,1,1} - X_{2,2,1}) \quad (3.89)$$

Adaptation

$$\left| f^{c}_{1,2,1,2,1} - 10(\theta_{1,1,2,1} - \theta_{2,1,2,1}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,2} - X_{1,2,1}) \quad (3.90)$$

$$\left| f^{c}_{2,3,1,2,1} - 6.67(\theta_{2,1,2,1} - \theta_{3,1,2,1}) \right| \le M(1 - X_{2,0} - X_{2,1} - X_{2,2} - X_{2,2,1}) \quad (3.91)$$

The line-flow constraints for candidate lines in the 2nd operating condition and for year 2 in scenario 1:

SP

$$\left| f^{c}_{1,2,2,2,1} - 10(\theta_{1,2,2,1} - \theta_{2,2,2,1}) \right| \le M(1 - X_{1,0} - X_{1,1,1} - X_{1,2,1}) \quad (3.92)$$

$$f^{c}_{2,3,2,2,1} - 6.67(\theta_{2,2,2,1} - \theta_{3,2,2,1}) \le M\left(1 - X_{2,0} - X_{2,1,1} - X_{2,2,1}\right) \quad (3.93)$$

Adaptation

$$\left| f^{c}_{1,2,2,2,1} - 10(\theta_{1,2,2,1} - \theta_{2,2,2,1}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,2} - X_{1,2,1}) \quad (3.94)$$

$$\left| f^{c}_{2,3,2,2,1} - 6.67(\theta_{2,2,2,1} - \theta_{3,2,2,1}) \right| \le M(1 - X_{2,0} - X_{2,1} - X_{2,2} - X_{2,2,1}) \quad (3.95)$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 3 in scenario 1:

SP

$$\left| f^{c}_{1,2,1,3,1} - 10(\theta_{1,1,3,1} - \theta_{2,1,3,1}) \right| \le M(1 - X_{1,0} - X_{1,1,1} - X_{1,2,1} - X_{1,3,1}) \quad (3.96)$$

$$\left| f^{c}_{2,3,1,3,1} - 6.67(\theta_{2,1,3,1} - \theta_{3,1,3,1}) \right| \le M(1 - X_{2,0} - X_{2,1,1} - X_{2,2,1} - X_{2,3,1}) \quad (3.97)$$

Adaptation

$$\left| f^{c}_{1,2,1,3,1} - 10(\theta_{1,1,3,1} - \theta_{2,1,3,1}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,2} - X_{1,3} - X_{1,3,1}) \quad (3.98)$$

$$\left| f^{c}_{2,3,1,3,1} - 6.67(\theta_{2,1,3,1} - \theta_{3,1,3,1}) \right| \le M(1 - X_{2,0} - X_{2,1} - X_{2,2} - X_{2,3} - X_{2,3,1}) \quad (3.99)$$

The line-flow constraints for candidate lines in the 2nd operating condition and for year 3 in scenario 1:

SP

$$\left| f^{c}_{1,2,2,3,1} - 10(\theta_{1,2,3,1} - \theta_{2,2,3,1}) \right| \le M(1 - X_{1,0} - X_{1,1,1} - X_{1,2,1} - X_{1,3,1}) \quad (3.100)$$

$$\left| f^{c}_{2,3,2,3,1} - 6.67(\theta_{2,2,3,1} - \theta_{3,2,3,1}) \right| \le M(1 - X_{2,0} - X_{2,1,1} - X_{2,2,1} - X_{2,3,1}) \quad (3.101)$$

Adaptation

$$\left| f^{c}_{1,2,2,3,1} - 10(\theta_{1,2,3,1} - \theta_{2,2,3,1}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,2} - X_{1,3} - X_{1,3,1}) \quad (3.102)$$

$$\left| f^{c}_{2,3,2,3,1} - 6.67(\theta_{2,2,3,1} - \theta_{3,2,3,1}) \right| \leq M(1 - X_{2,0} - X_{2,1} - X_{2,2} - X_{2,3} - X_{2,3,1}) \quad (3.103)$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 1 in scenario 2:

SP

$$\left| f^{c}_{1,2,1,1,2} - 10(\theta_{1,1,1,2} - \theta_{2,1,1,2}) \right| \leq M(1 - X_{1,0} - X_{1,1,2}) \quad (3.104)$$

$$\left| f^{c}_{2,3,1,1,2} - 6.67(\theta_{2,1,1,2} - \theta_{3,1,1,2}) \right| \leq M(1 - X_{2,0} - X_{2,1,2}) \quad (3.105)$$

Adaptation

$$\left| f^{c}_{1,2,1,1,2} - 10(\theta_{1,1,1,2} - \theta_{2,1,1,2}) \right| \leq M(1 - X_{1,0} - X_{1,1} - X_{1,1,2}) \quad (3.106)$$

$$\left| f^{c}_{2,3,1,1,2} - 6.67(\theta_{2,1,1,2} - \theta_{3,1,1,2}) \right| \leq M(1 - X_{2,0} - X_{2,1} - X_{2,1,2}) \quad (3.107)$$

The line-flow constraints for candidate lines in the 2nd operating condition and for year 1 in scenario 2:

SP

$$\left| f^{c}_{1,2,2,1,2} - 10(\theta_{1,2,1,2} - \theta_{2,2,1,2}) \right| \leq M(1 - X_{1,0} - X_{1,1,2}) \quad (3.108)$$

$$\left| f^{c}_{2,3,2,1,2} - 10(\theta_{2,2,1,2} - \theta_{3,2,1,2}) \right| \leq M(1 - X_{2,0} - X_{2,1,2}) \quad (3.109)$$

Adaptation

$$\left| f^{c}_{1,2,2,1,2} - 10(\theta_{1,2,1,2} - \theta_{2,2,1,2}) \right| \leq M(1 - X_{1,0} - X_{1,1} - X_{1,1,2}) \quad (3.110)$$

$$\left| f^{c}_{2,3,2,1,2} - 10(\theta_{2,2,1,2} - \theta_{3,2,1,2}) \right| \leq M(1 - X_{2,0} - X_{2,1} - X_{2,1,2}) \quad (3.111)$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 2 in scenario 2:

SP

$$\left| f^{c}_{1,2,1,2,2} - 10(\theta_{1,1,2,2} - \theta_{2,1,2,2}) \right| \leq M(1 - X_{1,0} - X_{1,1,2} - X_{1,2,2}) \quad (3.112)$$

$$\left| f^{c}_{2,3,1,2,2} - 6.67(\theta_{2,1,2,2} - \theta_{3,1,2,2}) \right| \le M(1 - X_{2,0} - X_{2,1,2} - X_{2,2,2}) \quad (3.113)$$

Adaptation

$$\left| f^{c}_{1,2,1,2,2} - 10(\theta_{1,1,2,2} - \theta_{2,1,2,2}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,2} - X_{1,2,2}) \quad (3.114)$$

$$\left| f^{c}_{2,3,1,2,2} - 6.67(\theta_{2,1,2,2} - \theta_{3,1,2,2}) \right| \le M(1 - X_{2,0} - X_{2,1} - X_{2,2} - X_{2,2,2}) \quad (3.115)$$

The line-flow constraints for candidate lines in the 2nd operating condition and for year 2 in scenario 2:

SP

$$\left| f^{c}_{1,2,2,2,2} - 10(\theta_{1,2,2,2} - \theta_{2,2,2,2}) \right| \le M(1 - X_{1,0} - X_{1,1,2} - X_{1,2,2}) \quad (3.116)$$

$$\left| f^{c}_{2,3,2,2,2} - 6.67(\theta_{2,2,2,2} - \theta_{3,2,2,2}) \right| \le M(1 - X_{2,0} - X_{2,1,2} - X_{2,2,2}) \quad (3.117)$$

Adaptation

$$\left| f^{c}_{1,2,2,2,2} - 10(\theta_{1,2,2,2} - \theta_{2,2,2,2}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,2} - X_{1,2,2}) \quad (3.118)$$

$$\left| f^{c}_{2,3,2,2,2} - 6.67(\theta_{2,2,2,2} - \theta_{3,2,2,2}) \right| \le M(1 - X_{2,0} - X_{2,1} - X_{2,2} - X_{2,2,2}) \quad (3.119)$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 3 in scenario 2:

SP

$$\left| f^{c}_{1,2,1,3,2} - 10(\theta_{1,1,3,2} - \theta_{2,1,3,2}) \right| \le M(1 - X_{1,0} - X_{1,1,2} - X_{1,2,2} - X_{1,3,2}) \quad (3.120)$$

$$\left| f^{c}_{2,3,1,3,2} - 6.67(\theta_{2,1,3,2} - \theta_{3,1,3,2}) \right| \le M(1 - X_{2,0} - X_{2,1,2} - X_{2,2,2} - X_{2,3,2}) \quad (3.121)$$

Adaptation

$$\left| f^{c}_{1,2,1,3,2} - 10(\theta_{1,1,3,2} - \theta_{2,1,3,2}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,2} - X_{1,3} - X_{1,3,2}) \quad (3.122)$$

$$\left| f^{c}_{1,2,1,3,2} - 10(\theta_{1,1,3,2} - \theta_{2,1,3,2}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,2} - X_{1,3} - X_{1,3,2}) \quad (3.123)$$

The line-flow constraints for candidate lines in the 2nd operating condition and for year 3 in scenario 2:

SP

$$\left| f^{c}_{1,2,2,3,2} - 10(\theta_{1,2,3,2} - \theta_{2,2,3,2}) \right| \le M(1 - X_{1,0} - X_{1,1,2} - X_{1,2,2} - X_{1,3,2}) \quad (3.124)$$

$$\left| f^{c}_{2,3,2,3,2} - 6.67(\theta_{2,2,3,2} - \theta_{3,2,3,2}) \right| \le M(1 - X_{2,0} - X_{2,1,2} - X_{2,2,2} - X_{2,3,2}) \quad (3.125)$$

Adaptation

$$\left| f^{c}_{1,2,2,3,2} - 10(\theta_{1,2,3,2} - \theta_{2,2,3,2}) \right| \le M(1 - X_{1,0} - X_{1,1} - X_{1,2} - X_{1,3} - X_{1,3,2}) \quad (3.126)$$

$$\left| f^{c}_{2,3,2,3,2} - 6.67(\theta_{2,2,3,2} - \theta_{3,2,3,2}) \right| \le M(1 - X_{2,0} - X_{2,1} - X_{2,2} - X_{2,3} - X_{2,3,2}) \quad (3.127)$$

The third set of constraints are line-flow constraints for candidate line for the whole planning horizon.

The number of these inequality constraints can be computed using the formula described below.

The 2 in the formula is because of the absolute value on the constraints.

=2*B*C*T*W

=2*2*3*2*2

=48

SP and adaptation have equal number of these types of constraints but they are formulated differently.

4) Capacity limit constraints for candidate lines

The line-flow constraints for candidate lines in the 1st operating condition and for year 1 in scenario 1:

SP

$$\left| f^{c}_{1,2,1,1,1} \right| \le f_{1,2,\max}(X_{1,0} + X_{1,1,1}) \quad (3.128)$$

$$\left| f^{c}_{2,3,1,1,1} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1,1}) \tag{3.129}$$

Adaptation

$$\left| f^{c}_{1,2,1,1,1} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,1,1}) \tag{3.130}$$

$$\left| f^{c}_{2,3,1,1,1} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1} + X_{2,1,1}) \tag{3.131}$$

The line-flow constraints for candidate lines in the 2nd operating condition and for year 1 in scenario 1:

SP

$$\left| f^{c}_{1,2,2,1,1} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1,1}) \tag{3.132}$$

$$\left| f^{c}_{2,3,2,1,1} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1,1}) \tag{3.133}$$

Adaptation

$$\left| f^{c}_{1,2,2,1,1} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,1,1}) \tag{3.134}$$

$$\left| f^{c}_{2,3,2,1,1} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1} + X_{2,1,1}) \tag{3.135}$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 2 in scenario 1:

SP

$$\left| f^{c}_{1,2,1,2,1} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1,1} + X_{1,2,1}) \tag{3.136}$$

$$\left| f^{c}_{2,3,1,2,1} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1,1} + X_{2,2,1}) \tag{3.137}$$

Adaptation

$$\left| f^{c}_{1,2,1,2,1} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,2} + X_{1,2,1}) \tag{3.138}$$

$$\left| f^{c}_{2,3,1,2,1} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1} + X_{2,2} + X_{2,2,1}) \tag{3.139}$$

The line-flow constraints for candidate lines in the 2nd operating condition and for year 2 in scenario 1:

SP

$$\left| f^{c}_{1,2,2,2,1} \right| \le f_{1,2,\max}(X_{1,0} + X_{1,1,1} + X_{1,2,1}) \quad (3.140)$$

$$\left| f^{c}_{2,3,2,2,1} \right| \le f_{2,3,\max}(X_{2,0} + X_{2,1,1} + X_{2,2,1}) \quad (3.141)$$

Adaptation

$$\left| f^{c}_{1,2,2,2,1} \right| \le f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,2} + X_{1,2,1}) \quad (3.142)$$

$$\left| f^{c}_{2,3,2,2,1} \right| \le f_{2,3,\max}(X_{2,0} + X_{2,1} + X_{2,2} + X_{2,2,1}) \quad (3.143)$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 3 in scenario 1:

SP

$$\left| f^{c}_{1,2,1,3,1} \right| \le f_{1,2,\max}(X_{1,0} + X_{1,1,1} + X_{1,2,1} + X_{1,3,1}) \quad (3.144)$$

$$\left| f^{c}_{2,3,1,3,1} \right| \le f_{2,3,\max}(X_{2,0} + X_{2,1,1} + X_{2,2,1} + X_{2,3,1})$$

Adaptation

$$\left| f^{c}_{1,2,1,3,1} \right| \le f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,2} + X_{1,3} + X_{1,3,1}) \quad (3.145)$$

$$\left| f^{c}_{2,3,1,3,1} \right| \le f_{2,3,\max}(X_{2,0} + X_{2,1} + X_{2,2} + X_{2,3} + X_{2,3,1}) \quad (3.146)$$

The line-flow constraints for candidate lines in the 2nd operating condition and for year 3 in scenario 1:

SP

$$\left| f^{c}_{1,2,2,3,1} \right| \le f_{1,2,\max}(X_{1,0} + X_{1,1,1} + X_{1,2,1} + X_{1,3,1}) \quad (3.147)$$

$$\left| f^{c}_{2,3,2,3,1} \right| \le f_{2,3,\max}(X_{2,0} + X_{2,1,1} + X_{2,2,1} + X_{2,3,1}) \quad (3.148)$$

Adaptation

$$\left| f^{c}_{1,2,2,3,1} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,2} + X_{1,3} + X_{1,3,1}) \tag{3.149}$$

$$\left| f^{c}_{2,3,2,3,1} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1} + X_{2,2} + X_{2,3} + X_{2,3,1}) \tag{3.150}$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 1 in scenario 2:

SP

$$\left| f^{c}_{1,2,1,1,2} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1,2}) \tag{3.151}$$

$$\left| f^{c}_{2,3,1,1,2} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1,2}) \tag{3.152}$$

Adaptation

$$\left| f^{c}_{1,2,1,1,2} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,1,2}) \tag{3.153}$$

$$\left| f^{c}_{2,3,1,1,2} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1} + X_{2,1,2}) \tag{3.154}$$

The line-flow constraints for candidate lines in the 2nd operating condition and for year 1 in scenario 2:

SP

$$\left| f^{c}_{1,2,2,1,2} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1,2}) \tag{3.155}$$

$$\left| f^{c}_{2,3,2,1,2} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1,2}) \tag{3.156}$$

Adaptation

$$\left| f^{c}_{1,2,2,1,2} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,1,2}) \tag{3.157}$$

$$\left| f^{c}_{2,3,2,1,2} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1} + X_{2,1,2}) \tag{3.158}$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 2 in scenario 2:

SP

$$\left| f^{c}_{1,2,1,2,2} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1,2} + X_{1,2,2}) \tag{3.159}$$

$$\left| f^{c}_{2,3,1,2,2} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1,2} + X_{2,2,2}) \tag{3.160}$$

Adaptation

$$\left| f^{c}_{1,2,1,2,2} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,2} + X_{1,2,2}) \tag{3.161}$$

$$\left| f^{c}_{2,3,1,2,2} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1} + X_{2,2} + X_{2,2,2}) \tag{3.162}$$

The line-flow constraints for candidate lines in the 1st operating condition and for year 3 in scenario 2:

SP

$$\left| f^{c}_{1,2,1,3,2} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1,2} + X_{1,2,2} + X_{1,3,2}) \tag{3.163}$$

$$\left| f^{c}_{2,3,1,3,2} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1,2} + X_{2,2,2} + X_{2,3,2}) \tag{3.164}$$

Adaptation

$$\left| f^{c}_{1,2,1,3,2} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,2} + X_{1,3} + X_{1,3,2}) \tag{3.165}$$

$$\left| f^{c}_{1,2,1,3,2} \right| \leq f_{2,3,\max}(X_{1,0} + X_{1,1} + X_{1,2} + X_{1,3} + X_{1,3,2}) \tag{3.166}$$

The line-flow constraints for candidate lines in the 2nd operating condition and for year 3 in scenario 2:

$$\left| f^{c}_{1,2,2,3,2} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1,2} + X_{1,2,2} + X_{1,3,2}) \tag{3.167}$$

$$\left| f^{c}_{2,3,2,3,2} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1,2} + X_{2,2,2} + X_{2,3,2}) \tag{3.168}$$

Adaptation

$$\left| f^{c}_{1,2,2,3,2} \right| \leq f_{1,2,\max}(X_{1,0} + X_{1,1} + X_{1,2} + X_{1,3} + X_{1,3,2}) \tag{3.169}$$

$$\left| f^{c}_{2,3,2,3,2} \right| \leq f_{2,3,\max}(X_{2,0} + X_{2,1} + X_{2,2} + X_{2,3} + X_{2,3,2}) \tag{3.170}$$

The fourth set of constraints are capacity limits for candidate lines for the whole planning horizon.

The number of these inequality constraints can be computed using the formula described below. The 2 in the formula is because of the absolute value on the constraints.

=2*B*C*T*W

=2*2*3*2*2

=48

SP and adaptation have equal number of these types of constraints but they are formulated differently.

5.) **Special constraints unique to both approaches**

SP

A transmission candidate can only be invested in a scenario once (1st candidate):

$$X_{1,0} + X_{1,1,1} + X_{1,2,1} + X_{1,3,1} \leq 1 \quad (3.171)$$

$$X_{1,0} + X_{1,1,2} + X_{1,2,2} + X_{1,3,2} \leq 1 \quad (3.172)$$

A transmission candidate can only be invested in a scenario once (2nd candidate):

$$X_{2,0} + X_{2,1,1} + X_{2,2,1} + X_{2,3,1} \leq 1 \quad (3.173)$$

$$X_{2,0} + X_{2,1,2} + X_{2,2,2} + X_{2,3,2} \leq 1 \quad (3.174)$$

Adaptation

Special constraints (Number 1)

This constraints ensures that the 1st candidate can only be invested in the trajectory once:

$$X_{1,0} + X_{1,1} + X_{1,2} + X_{1,3} \leq 1 \quad (3.175)$$

This constraints ensures that the 2nd candidate can only be invested in the trajectory once

$$X_{2,0} + X_{2,1} + X_{2,2} + X_{2,3} \leq 1 \quad (3.176)$$

Special constraints (Number 2)

This constraints ensures that if candidate is invested in the core-trajectory it is not available to be adapted to a scenario (i.e. for the 1st candidate and the 1st scenario):

$$X_{1,0} + X_{1,1} + X_{1,1,1} \leq 1 \quad (3.177)$$

$$X_{1,0} + X_{1,1} + X_{1,2} + X_{1,2,1} \leq 1 \quad (3.178)$$

$$X_{1,0} + X_{1,1} + X_{1,2} + X_{1,3} + X_{1,3,1} \leq 1 \quad (3.179)$$

This constraints ensures that if candidate is invested in the core-trajectory it is not available to be adapted to a scenario (i.e. for the 1st candidate and the 2nd scenario):

$$X_{1,0} + X_{1,1} + X_{1,1,2} \leq 1 \quad (3.180)$$

$$X_{1,0} + X_{1,1} + X_{1,2} + X_{1,2,2} \leq 1 \quad (3.181)$$

$$X_{1,0} + X_{1,1} + X_{1,2} + X_{1,3} + X_{1,3,2} \leq 1 \quad (3.182)$$

This constraints ensures that if candidate is invested in the core-trajectory it is not available to be adapted to a scenario (i.e. for the 2nd candidate and the 1st scenario):

$$X_{2,0} + X_{2,1} + X_{1,1,1} \leq 1 \quad (3.183)$$

$$X_{2,0} + X_{2,1} + X_{2,2} + X_{1,2,1} \leq 1 \quad (3.184)$$

$$X_{2,0} + X_{2,1} + X_{2,2} + X_{2,3} + X_{1,3,1} \leq 1 \quad (3.185)$$

This constraints ensures that if candidate is invested in the core-trajectory it is not available to be adapted to a scenario (i.e. for the 2nd candidate and the 2nd scenario):

$$X_{2,0} + X_{2,1} + X_{1,1,2} \leq 1 \tag{3.186}$$

$$X_{2,0} + X_{2,1} + X_{2,2} + X_{1,2,2} \leq 1 \tag{3.187}$$

$$X_{2,0} + X_{2,1} + X_{2,2} + X_{2,3} + X_{1,3,2} \leq 1 \tag{3.188}$$

SP

Special constraints

= B*W

= 2*2

=4

Adaptation

Special constraints 1

=B

=2

Special constraints 2

=B*T*W

=2*3*2

=12

Total number of constraints

SP

= NB*C*T*W+E*C*T*W+2*B*C*T*W+2*B*C*T*W+ B*W

Adaptation

$= NB*C*T*W+E*C*T*W+2*B*C*T*W+2*B*C*T*W+ B+ B*T*W$

Total number of variables

The number of integer variables

SP

$TB_{sp}= B*(1+(T)*W)$

Adaptation

$TB_{a}= B*(1+ (T)*(W+1))$

The factor W+1 is because of the core-trajectory

The number of continuous variables are

SP

$TC_{sp} = (E+B)*C*T*W$

Adaptation

$TC_{a} = (E+B)*C*T*W$

The reason this study is useful is that it shows how these two approaches differ in different types of constraints unique to transmission planning. It was found that these approaches differ in three types of constraints, which are line-flow constraints for candidate lines, capacity limit constraints for candidate lines and special constraints unique to both approaches. They both have the same number of constraints apart from the number of special constraints unique to both approaches. The number of special constraints are not large in number, so they will necessary increase the computational complexity for both approaches. In terms of variables, the only difference is the extra binary variable for the core-trajectory of adaptation.

CHAPTER 4. ADAPTATION FORMULATION AND PROCEDURE

This chapter describes the mathematical formulation of the adaptation for different types of planning problems. The chapter also describes the procedures involved in planning using adaptation. Finally, the software design process is also described.

4.1 Formulation

This section describes the mathematical formulation for three types of planning problem. They are generation expansion planning, transmission expansion planning and co-optimization of both generation and transmission resources.

4.1.1 Generation planning

Nomenclature

$OM^{v}_{k,t}$ is the variable O&M costs of generator k, at time t

$OM^{F}_{k,t}$ is the fixed O&M costs of generator k, at time t

$FC_{k,t,w}$ is the fuel cost of generator k, at time t in scenario w

M is a large number

c is the number of operating conditions in a year

$f_{c,t,w}$ is the vector of line-flows in operating condition c, at time t, in scenario w

$P_{c,t,w}$ is the vector of dispatched power in operating condition c, at time t, in scenario w

$D_{c,t,w}$ is the vector of demand for operating condition c, at time t, in scenario w

S is the node-arc incidence matrix

K is the generator index

β is a trade-off parameter

T is the planning horizon

w designates the scenario

h_c is the number of hours in operating condition c

$I_{k,t}$ is the investment cost of generator k per MW at time t

$Cap^{f}_{k,t}$ is the core-investment trajectory of generation at bus k at time t

$Cap^{f}_{k,t-1}$ is the core-investment trajectory of generation at bus k at time t-1

CF_k is capacity factor of generator k

$\Delta Cap_{k,t,w}$ is the additional capacity needed to adapt to scenario w at bus k at time t

$Cap^{add}_{k,t}$ is the additional capacity added to core investment trajectory at bus k at time t

$Cap_{k,t,w}$ is the capacity at bus k at time t at scenario w

$P_{k,t,c,w}$ is the dispatched power of generator k at operating condition c at time t and scenario w

$\gamma_{i,j}$ is the element (i,j) in the susceptance matrix

$f_{i,j,c,t,w}$ is the line-flow from bus i to bus j in operating condition c at time t in scenario w for a candidate line

$f_{i,j,max}$ is the maximum capacity of a candidate line from bus i to bus j

$Cap^{ret}_{k,t}$ is the capacity of retired generation at bus k at time t

$\theta_{i,c,t,w}$ is the angle at bus i at operating condition at time t in scenario w

$\theta_{j,c,t,w}$ is the angle at bus j at operating condition at time t in scenario w

$$Min \sum_{t=1}^{T}\sum_{k=1}^{K} I_{k,t} Cap^{add}_{k,t} + \sum_{w=1}^{W}\sum_{t=1}^{T}\sum_{k=1}^{K} OM^{F}_{k,t} Cap_{k,t,w}$$
$$+\sum_{w=1}^{W}\sum_{t=1}^{T}\sum_{k=1}^{K}(\sum_{c=1}^{C}(OM^{V}_{k,t} + FC_{k,t,w}))P_{k,t,c,w} h_c + \beta \sum_{w=1}^{W}\sum_{t=1}^{T}\sum_{k=1}^{K} \Delta Cap_{k,t,w} \tag{4.1}$$

Subject to

$$sf_{c,t,w} + P_{c,t,w} - D_{c,t,w} = 0 \tag{4.2}$$

$$f_{i,j,c,t,w} - \gamma_{ij}(\theta_{i,c,t,w} - \theta_{j,c,t,w}) = 0 \tag{4.3}$$

$$Cap_{k,t}^{f} = Cap_{k,t-1}^{f} + Cap_{k,t}^{add} - Cap_{k,t}^{ret} \tag{4.4}$$

$$Cap_{k,t,w} = Cap_{k,t}^{f} + \Delta Cap_{k,t,w} \tag{4.5}$$

$$0 \leq P_{k,t,c,w} \leq CF_{k} * Cap_{k,t,w} \tag{4.6}$$

$$Cap_{k,t}^{f} \geq 0 \tag{4.7}$$

$$Cap_{k,t}^{add} \geq 0 \tag{4.8}$$

$$\Delta Cap_{k,t,w} \geq 0 \tag{4.9}$$

The first term in the objective function is the additional investment for the core trajectory investment update and the salvage value is subtracted from the investment cost. The second term in the objective function is the fixed O&M costs for all generators. The third term in the objective function is the sum of variable O&M costs for the generators and the fuel costs for the generators. The fourth term in the objective is additional capacity needed to adapt to future scenarios at each time/stage. Discount factors is applied to all the costs.

The fourth term is multiplied by β which is the trade-off parameter. When β is high, the core- trajectory investment is high and adaptation is low, while when β is low, core-trajectory investment is low and adaptation is high. β has to be well selected in order not to be at both extremes.

Equation (4.2) represents the power demand balance equation for all given operating conditions in a year, years in the planning period and all scenarios in the model. Equation (4.3) represents line flow constraints for existing lines for all given operating conditions in a year, all the years in the planning period and all scenarios in the model. Equation (4.4) is the core-trajectory update equation for generation investment. Equation (4.5) represents adaptation from the core-trajectory to different scenarios. Equation (4.6) is the allowable power a generator is allowed to dispatch in an operating condition. While the remaining equations (4.7, 4, 8, and 4.9) signifies that the decision variables involved have to be non-negative.

4.1.2 Transmission planning

The Transmission Expansion Problem objective is to identify which transmission lines to build, where to build them, the capacity of line to build and when to build the lines.

MATHEMATICAL MODEL FOR TRANSMISSION PLANIING

A classical transmission expansion problem can be formulated as follows [43]

$$\text{Min} \sum_{(i,j)} c_{ij} x_{ij} \tag{4.10}$$

Subject to

$$s.f + g = d \tag{4.11}$$

$$f_{ij}^{0} - \gamma_{ij}^{0}(\theta_i - \theta_j) = 0 \tag{4.12}$$

$$f_{ij} - x_{ij}\gamma_{ij}(\theta_i - \theta_j) = 0 \tag{4.13}$$

$$\left| f_{ij}^{0} \right| \leq f_{ij}^{0,\max} \tag{4.14}$$

$$\left| f_{ij} \right| \leq x_{ij} f_{ij}^{\max} \tag{4.15}$$

$$x_{ij} \text{ Integer} \tag{4.16}$$

where c_{ij} is the cost of transmission line that goes from bus i to bus j, x_{ij} is the binary variable for the transmission candidate line that goes from bus i to bus j, s is node-arc incidence matrix, f is the vector of line-flows, and g is the vector of dispatched generation and d is the vector of demand. f_{ij}^0 is the line-flow for existing line and f_{ij} is the line-flow for candidate transmission lines.

Big-M formulation

Transmission expansion planning is a mixed-integer non-linear optimization problem [44]. Due to nonlinearity in constraints (i.e.2nd Kirchhoff's law for candidate lines) as a result of multiplication of candidate susceptance and angle differences, nonlinearities are transformed by introducing a disjoint mixed integer constraint with parameter Big "M"[45]. The big-M approach can be extended to a multi-stage/multi-period planning problem by using two approaches.

Method 1

The single stage disjunctive model can be transformed to the following equation by introducing big-M

$$\left| f_{ij} - \gamma_{ij}(\theta_i - \theta_j) \right| \leq M(1 - X_j) \tag{4.17}$$

$$\left| f_{ij} \right| \leq f_{ij,\max} X_j \tag{4.18}$$

The equations above can be transformed into a multi-stage formulation by using the following equations [46]

$$\left| f_{ijt} - \gamma_{ij}(\theta_{it} - \theta_{jt}) \right| \leq M(1 - \sum_{t=1}^{T} X_{jt}) \tag{4.19}$$

$$\left| f_{ijt} \right| \leq f_{ij,\max} \sum_{t=1}^{T} X_{jt} \tag{4.20}$$

$$\sum_{t=1}^{T} X_{jt} \leq 1 \tag{4.21}$$

The second approach differs from the first approach in the way that you don't have to repeat previous binary investment decision for previous time stages at a future time stage in the line flow constraints for candidate lines. The line-flow constraints for transmission candidates just contain one binary investment decision [47].

Mixed integer linear programs

Mixed integer programs are optimization programs with mixed decision variables (i.e. integer and continuous variables). Mixed integer programs are NP-Hard problems and sometimes very difficult to solve. Transmission expansion planning problems are a special case of MILPs called binary mixed integer programming, since the binary variables takes either 0 or 1. A standard mixed integer program can be formulated as follows.

$$\min_{x,z} \; f_1^T x + f_2^T z \tag{4.22}$$

$$A_1 x + A_2 x \tag{4.23}$$

$$z \text{ Integer} \tag{4.24}$$

Methods for solving mixed integer linear problems (MILPs)

There are several methods for solving mixed integer problems. This sections describes various known approaches and also discusses their strength and weaknesses.

1) Branch and Bound

2) Cutting Plane

3) Branch and Cut

4) Heuristics

Branch and Bound

The first step in B&B is to solve the LP relaxation problem, by relaxation we mean that the integer constraints are converted to continuous variables. This algorithm develops a tree based on LP relaxation and explores the branches of the tree.

Cutting Plane

The cutting plane is a well-known approach for solving MILPs. The idea behind the cutting plane approach is to iteratively add cut (i.e. linear inequalities) to a linear constraints of an LP until the optimal basic feasible solution becomes integer.

Branch and cut

This approach is the combination of B&B and the cutting plane method.

Heuristics

The disadvantage with the heuristics is that it finds approximate solutions and not optimal solutions. However, heuristics tend to be faster than algorithms that can solve for the optimal solution. Therefore a trade-off needs to be established between accuracy and time required to solve the MILP problem. An example of this method's application can be found in [48].

Mathematical formulation of adaptation

Nomenclature

S is the node-arc incidence matrix

K is the generator index

Beta is a trade-off parameter

T is the planning horizon

w designates the scenario

IC_j is the investment cost of the jth transmission line candidate

h_c is the number of hours in operating condition c

D is decision stages

$OM^v_{k,t}$ O&M costs of generator k, at time t

$FC_{k,t,w}$ is the fuel cost of generator k, at time t in scenario w

$P_{k,t,c,w}$ is the dispatched power of generator k, at operating condition c at time t in scenario w

$X_{j,d(t)}$ is the binary transmission candidate at decision stage d as a function of time

$X_{j,d(t),w}$ is the binary transmission candidate at decision stage d in scenario w

M is a large number

C is the number of operating conditions in a year

$f_{c,t,w}$ is the vector of line-flows in operating condition c, at time t, in scenario w

$P_{c,t,w}$ is the vector of dispatched power in operating condition c, at time t, in scenario w

$D_{c,t,w}$ is the vector of demand for operating condition c, at time t, in scenario w

$\gamma_{i,j}$ is the element (i,j) in the susceptance matrix

d(t) is decision stage as a function of time

d(T) is the end of the planning horizon

$fi_{,j,c,t,w}$ is the line-flow from bus i to bus j in operating condition c at time t in scenario w for a candidate line

$f_{i,j,c,t,w}^0$ is the line-flow from bus i to bus j in operating condition c at time t in scenario w for a candidate line

$f_{i,j,max}^0$ is the maximum capacity of an existing line from bus i to bus j

$f_{i,j,max}$ is the maximum capacity of a candidate line from bus i to bus j

$\theta_{i,c,t,w}$ is the angle at bus i at operating condition c at time t in scenario w

$\theta_{j,c,t,w}$ is the angle at bus j at operating condition c at time t in scenario w

$$Min \sum_{d=1}^{D}\sum_{j=1}^{J} IC_{j,d(t)} X_{j,d(t)} + \sum_{w=1}^{W}\sum_{t=1}^{T}\sum_{k=1}^{K}(\sum_{c=1}^{C}(OM_{k,t}^{V} + FC_{k,t,w}))P_{k,t,c,w}h_c + \beta\sum_{w=1}^{W}\sum_{j=1}^{J}\sum_{d=2}^{D} IC_j X_{j,d(t),w} \tag{4.25}$$

Subject to

$$sf_{c,t,w} + P_{c,t,w} - D_{c,t,w} = 0 \tag{4.26}$$

$$f_{i,j,c,t,w}^{0} - \gamma_{ij}(\theta_{i,c,t,w} - \theta_{j,c,t,w}) = 0 \tag{4.27}$$

$$\left| f_{i,j,c,t,w} - \gamma_{ij}(\theta_{i,c,t,w} - \theta_{j,c,t,w}) \right| \le M(1 - \sum_{d=1}^{D} X_{j,d(t)}[d(t), d(T)] - \sum_{w=1}^{W}\sum_{d=D}^{D} X_{j,d(t),w}[d(t), d(t+1)]) \quad \forall w, d, t \tag{4.28}$$

$$\left| f_{i,j,c,t,w}^{0} \right| \le f_{i,j,\max}^{0} \tag{4.29}$$

$$\left| f_{i,j,c,t,w} \right| \le f_{i,j,\max}(\sum_{d=1}^{D} X_{j,d(t)}[d(t), d(T)] + \sum_{w=1}^{W}\sum_{d=D}^{D} X_{j,d(t),w}[d(t), d(t+1)]) \quad \forall w, d, t \tag{4.30}$$

$$X_{j,d(t)} \in \{0,1\} \tag{4.31}$$

$$X_{j,d(t),w} \in \{0,1\} \tag{4.32}$$

This constraints ensures that no candidate is invested twice in the core-trajectory solution

$$\sum_{d=1}^{D} X_{j,d(t)} \le 1 \quad \forall j \tag{4.33}$$

This constraints ensure that at a particular stage if a transmission is invested in the core-trajectory, it is not available to be adapted to scenarios.

$$X_{j,d(1)} + \sum_{d=2}^{D} X_{j,d(t)} + \sum_{d=D}^{D} \sum_{w=1}^{W} X_{j,d(t),w} \quad \forall j, D \geq 2 \tag{4.34}$$

In the objective function in eqn (4.25), the first term represents the cost of the core – trajectory for transmission investment, the second term is the costs of dispatched generation under all considered scenarios. The third term represents the costs of adaptive transmission investment under a given scenario. Discount factors is applied to all the costs.

Equation (4.26) represents the power demand balance equation for all given operating conditions in a year, all the years in the planning horizon and all scenarios in the model. Equation (4.27) represents line flow constraints for existing lines for all given operating conditions in a year, all the years in the planning horizon and all scenarios in the model. . Equation (4.28) represents line flow constraints for candidate lines for all given operating conditions in a year, all the years in the planning horizon and all scenarios in the model.

The first summation in equation (4.28) is the core trajectory for transmission investment. The investment decision for the core-trajectory is not assumed to be made every year but at designated decision stages. For instance, assuming a study has a planning horizon of 20 years and there are four decision stages (t=0 (i.e. the initial decision), 5^{th}, 10^{th} 15^{th} year). The transmission candidate for the initial decision is available from the beginning of year 1 to the end of the planning horizon (i.e. [d (0), d (20)]), likewise the transmission candidate at the 5^{th} year is available from the beginning of year 5 to the end of the planning horizon (i.e. [d (5), d (20)]). The second summation in equation (4.28) is the adaptable transmission investment. The adaptable transmission solution is assumed to be made at designated decision stages just like the core trajectory for transmission investment. However. adaptable transmission investment is formulated differently, because unlike the core transmission investment which is updated based on previous investment, investment decisions at a decision stage does not depend on the previous adapted investment in a particular scenario. At the initial stage there is no adaptation, so adaptation starts after uncertainty is revealed.

Equation (4.29) represents the capacity limit for all existing lines. Equation (4.30) represents the capacity limit for all candidate transmission lines.

Transportation model as a lower bound

Solving the transportation model of TEP is fast and serves as a lower bound to the solving the TEP model. We can exploit the solution of the transportation model to help solve our system faster.

$$Min \sum_{(i,j)} c_{ij} x_{ij} \tag{4.35}$$

Subject to

$$s.f + g = d \tag{4.36}$$

$$\left| f_{ij}^{0} \right| \leq f_{ij,\max}^{0} \tag{4.37}$$

$$\left| f_{ij} \right| \leq x_{ij} f_{ij,\max} \tag{4.38}$$

$$0 \leq g_i \leq g_{i,\max} \tag{4.39}$$

$$x_{ij} \text{ Integer} \tag{4.40}$$

where c_{ij} is the cost of transmission line that goes from bus i to bus j, x_{ij} is the binary variable for transmission candidate line that goes from bus i to bus j, s is node-arc incidence matrix, f is the vector of line-flows, g is the vector of dispatched generation and d is the vector of demand. f^0_{ij} is the line-flow for existing line and f_{ij} is the line-flow for candidate transmission lines.

4.1.3 Co-optimization

In tradition power system planning, Generation Expansion Planning (GEP) and Transmission Expansion Planning (TEP) are done separately. This has led to poor decision making due to ineffective co-ordination of both kind of expansion plans. This also leads to sub-optimal power system expansion decisions. Co-optimization is different from multi-objective optimization (or programming), also known as multi-criteria or multi-attribute optimization, which is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. According to [49] "Co-optimization is the simultaneous identification of two or more classes of investment decisions within one optimization strategy".

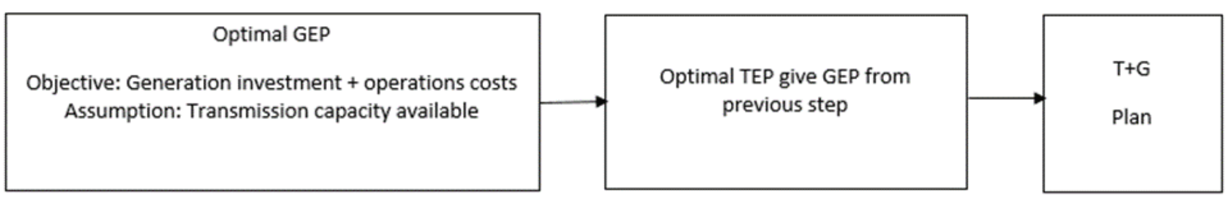

Figure 10: Traditional approach

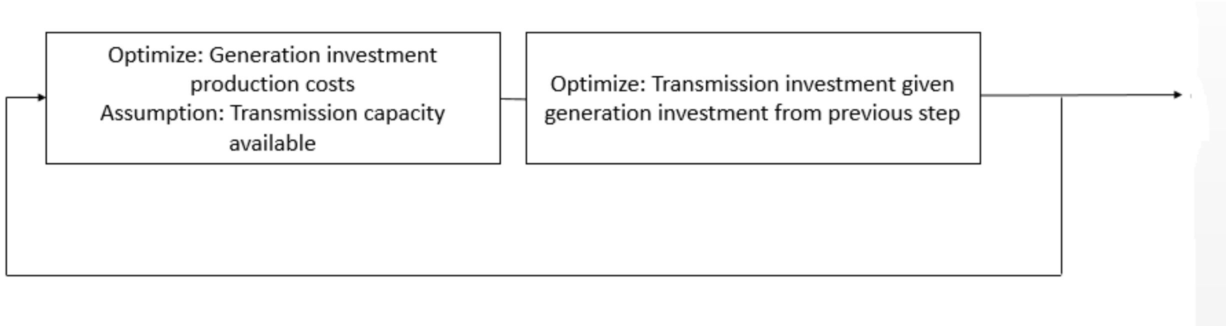

Figure 11: Better approach

Objective: Generation investment + Transmission investment + operations costs

Figure 12: Best approach

Recent application of co-optmization in power systems

Hedman et al. [50] applied to the concept of co-optimization to generation unit commitment and transmission switching considering N-1 reliability. The concept of transmission switching refers to the switching of lines in and out of the network to maximize economic benefits. The authors show the impact of transmission switching on optimal generation unit commitment. Co-optimization has been applied to power markets. Tan et al. [51] applied co-optimization to energy and reserves supplied by both demand and supply participants in an electric market .

However, it must be recognized that co-optimization, as a term used to refer to simultaneous identification of related decisions within a single optimization strategy, should not be understood to suggest any particular structural characteristics, at least not by virtue of their being co-optimization problems. Specifically, we may represent a co-optimization problem as:

Problem C:

$\min f(x,y)$

subject to

$g(x,y) \le b$

$h(x,y) = c$

The decision variables, x, and y, are related through the constraints g and h, i.e., one or more of the constraints contain both types of decision variables. Co-optimization means addressing this problem with both sets x and y remaining decision variables. In this sense, co-optimization is just the correct statement of problems that have heretofore been solved approximately, as follows:

Problem C-fixed:

$\min f(x,y^f)$

subject to

$g(x,y^f)<=b$

$h(x,y^f)=c$

where yf represents fixed values of y. When we solve Problem C-fixed instead of Problem C, we do so because Problem C is too computationally challenging to solve.

The point here is that the term "Co-optimization," when used to identify a problem we intend to solve, implies that we think we have the computational capabilities to solve the problem exactly (as Problem C) rather than approximately (as problem C-fixed).

Deterministic formulation of co-optimization

This section describes the mathematical model for co-optimization in a deterministic framework.

Nomenclature

S is the node-arc incidence matrix

K is the generator index

hc is the number of hours in operating condition c

D is the number of decision stages

$FC_{k,t}$ is the fuel cost of generator k, at time t

$X_{j,d(t)}$ is the binary transmission candidate at decision stage d as a function of time

M is a large number

c is operating conditions in a year

$f_{c,t}$ is the vector of line-flows in operating condition c, at time t

$P_{c,t}$ is the vector of dispatched power in operating condition c, at time t

$D_{c,t}$ is the vector of demand for operating condition c, at time t

$OM^{v}_{k,t}$ is the operation and maintenance costs of generator k, at time t

$Cap^{add}_{k,d(t)}$ is the additional capacity added at bus k at decision stage d(t)

$Cap^{k}_{,d(t)}$ is the capacity at bus k at decision stage d(t)

$$Min\sum_{d-1}^{D}\sum_{j=1}^{J} IC_{j,t} X_{j,d(t)} + \sum_{d=1}^{D}\sum_{k=1}^{K} IC_{k,t} Cap^{add}_{k,d(t)}$$
$$+\sum_{t=1}^{T}\sum_{k=1}^{K}\left(\sum_{c=1}^{C}(OM^{V}_{k,t} + FC_{k,t})\right) P_{k,t,c} h_c + \sum_{t=1}^{T}\sum_{k=1}^{K} OM^{F}_{k,t} Cap_{k,t} \tag{4.41}$$

Subject to

Transmission constraints

$$sf_{c.t} + P_{c,t} - D_{c,t} = 0 \tag{4.42}$$

$$f_{i,j,c,t} - \gamma_{ij}(\theta_{i,c,t} - \theta_{j,c,t}) = 0 \tag{4.43}$$

$$\left| f_{i,j,c,t} - \gamma_{ij}(\theta_{i,c,t} - \theta_{j,c,t}) \right| \le M\left(1 - \sum_{d=1}^{D} X_{j,d(t)}\right) \tag{4.44}$$

$$\left| f_{i,j,c,t} \right| \le f^{0}_{i,j,\max} \tag{4.45}$$

$$\left| f_{i,j,c,t} \right| \le f_{i,j,\max} \sum_{d=1}^{D} X_{j.d(t)} \tag{4.46}$$

$$X_{j,d(t)} \in \{0,1\} \quad (4.47)$$

This constraint ensures that a line is only built once

$$\sum_{d=1}^{D} X_{j.d(t)} \leq 1 \quad (4.48)$$

Generation constraints

$$Cap_{k,d(t)}^{add} \geq 0 \quad (4.49)$$

$$Cap_{k,d(t)} = Cap_{k,d(t-1)} + Cap_{k,d(t)}^{add} \quad (4.50)$$

$$0 \leq P_{k,t,c} \leq CF * Cap_{k,d(t)} \quad (4.51)$$

Mathematical formulation co-optimization using adaptation

This section describes the mathematical formulation of co-optimization of generation and transmission resources using adaptation. In this co-optimization formulation the transmission candidates decision variables are modelled as integer variables, while the generation decision variables are modelled as continuous variables

Nomenclature

S is the node-arc incidence matrix

K is the generator index

h_c is the number of hours in operation condition c

D is the number of decision stages

$FC_{k,t,w}$ is the fuel cost of generator k, at time t in scenario w

$X_{j,d(t)}$ is the binary transmission candidate at decision stage d as a function of time

$X_{j,d(t),w}$ is the binary transmission candidate at decision stage d as a function of time in scenario w

M is a large number

$IC_{j,d(t)}$ is the investment cost of transmission candidate j at time d(t)

$IC_{k,d(t)}$ is the investment cost of generation at bus k at time d(t)

$f_{c,t,w}$ is the vector of line-flows in operating condition c, at time t, in scenario w

$P_{c,t,w}$ is the vector of dispatched power in operating condition c, at time t, in scenario w

$D_{c,t,w}$ is the vector of demand for operating condition c, at time t, in scenario w

$\gamma_{i,j}$ is the element (i,j) in the susceptance matrix

d(t) is decision stage as a function of time

d(T) is the end of the planning horizon

$f_{i,j,c,t,w}$ is the line-flow from bus i to bus j in operating condition c at time t in scenario w for a candidate line

$f_{i,j,c,t,w}^{0}$ is the line-flow from bus i to bus j in operating condition c at time t in scenario w for an existing line

$f_{i,j,max}^{0}$ is the maximum capacity of an existing line from bus i to bus j

$f_{i,j,max}$ is the maximum capacity of a candidate line from bus i to bus j

$Cap^{f}_{k,d(t)}$ is the core-investment trajectory of generation at bus k at time d(t)

$Cap^{f}_{k,d(t-1)}$ is the core-investment trajectory of generation at bus k at time d(t-1)

CF_k is capacity factor of generator k

$\Delta Cap_{k,d(t),w}$ is the additional capacity needed to adapt to scenario w at bus k at time d(t)

$Cap^{add}_{k,d(t)}$ is the additional capacity added to core investment trajectory at bus k at time d(t)

$Cap_{k,d(t),w}$ is the capacity at bus k at time d(t) at scenario w

$P_{k,t,c,w}$ is the dispatched power of generator k at operating condition c at time t in scenario w

$\theta_{i,c,t,w}$ is the angle at bus i at operating condition at time t in scenario w

$\theta_{j,c,t,w}$ is the angle at bus j at operating condition at time t in scenario w

$$Min \sum_{d=1}^{D}\sum_{j=1}^{J} IC_{j,d(t)} X_{j,d(t)} + \sum_{d=1}^{D}\sum_{k=1}^{K} IC_{k,d(t)} Cap_{k,d(t)}^{add}$$
$$+ \sum_{w=1}^{W}\sum_{t=1}^{T}\sum_{k=1}^{K} OM_{k,t}^{F} Cap_{k,t,w} + \sum_{w=1}^{W}\sum_{t=1}^{T}\sum_{k=1}^{K} (\sum_{c=1}^{C} (OM_{k,t}^{V} + FC_{k,t,w})) P_{k,t,c,w} h_C \quad (4.52)$$
$$+ \beta \sum_{w=1}^{W}\sum_{j}^{J}\sum_{d=2}^{D} IC_{j,d(t)} X_{j,d(t),w} + \beta \sum_{w=1}^{W}\sum_{j=1}^{J}\sum_{d=2}^{D} IC_{j,d(t)} \Delta Cap_{k,d(t),w}$$

Subject to

$$sf_{c,t,w} + P_{c,t,w} - D_{c,t,w} = 0 \quad (4.53)$$

$$f_{i,j,c,t,w}^{0} - \gamma_{ij}(\theta_{i,c,t,w} - \theta_{j,c,t,w}) = 0 \quad (4.54)$$

$$f_{i,j,c,t,w} - \gamma_{ij}(\theta_{i,c,t,w} - \theta_{j,c,t,w}) \le M(1 - \sum_{d=1}^{D} X_{j,d(t)}[d(t), d(T)]$$
$$- \sum_{w=1}^{W}\sum_{d=D}^{D} X_{j,d(t),w}[d(t), d(t+1)]) \quad \forall w, d, t \quad (4.55)$$

$$\left| f_{i,j,c,t,w}^{0} \right| \le f_{i,j,\max}^{0} \quad (4.56)$$

$$\left| f_{i,j,c,t,w} \right| \le f_{i,j,\max} (\sum_{d=1}^{D} X_{j,d(t)}[d(t), d(T)]$$
$$+ \sum_{w=1}^{W}\sum_{d=D}^{D} X_{j,d(t),w}[d(t), d(t+1)]) \quad \forall w, d, t \quad (4.57)$$

$$Cap_{k,d(t)}^{f} = Cap_{k,d(t-1)}^{f} + Cap_{k,d(t)}^{add} - Cap_{k,d(t)}^{ret} \quad (4.58)$$

$$Cap_{k,d(t),w} = Cap^{f}_{k,d(t)} + \Delta Cap_{k,d(t),w} \quad (4.59)$$

$$Cap^{f}_{k,d(t)} \geq 0 \quad (4.60)$$

$$Cap^{add}_{k,d(t)} \geq 0 \quad (4.61)$$

$$X_{j,d(t)} \in \{0,1\} \quad (4.62)$$

$$X_{j,d(t),w} \in \{0,1\} \quad (4.63)$$

$$0 \leq P_{k,t,c,w} \leq CF * Cap_{k,d(t),w} \quad (4.64)$$

This constraints ensures that no candidate is invested twice in the core-trajectory solution.

$$\sum_{d=1}^{D} X_{j,d(t)} \leq 1 \quad \forall j \quad (4.65)$$

This constraints ensures that at a particular stage if a transmission is invested in the core-trajectory, it is not available to be adapted to scenarios.

$$X_{j,d(1)} + \sum_{d=2}^{D} X_{j,d(t)} + \sum_{d=D}^{D} \sum_{w=1}^{W} X_{j,d(t),w} \quad \forall j, D \geq 2 \quad (4.66)$$

In the objective function of the co-optimization formulation (i.e. 4.52), the first term is the core trajectory for transmission investment and it is updated throughout the planning horizon. The second term in the objective function is the additional investment that updates the core trajectory for generation investment and the salvage value is subtracted from the investment cost. The third second term is the fixed O&M costs for all generators. The fourth term in the objective function is the sum of variable O&M costs and the fuel costs for generators. The fifth term in the objective is additional transmission investment needed to adapt to future scenarios at each time stage. The sixth term in the objective is additional generation investment needed to adapt to future scenarios at each time stage. The discount factor is applied to all costs.

Equation 4.53 represents the power demand balance equation for all given operating conditions in a year, during the planning period over all scenarios in the problem. Equation 4.54 represents line flow constraints for existing lines for all given operating conditions in a year, all the years in the planning period and all scenarios in the model.

Equation 4.55 represents the line-flow constraint for candidate lines and M is a large number, after the inequality sign, the first term before the negative sign is core trajectory for transmission candidates while the second term is the transmission needed to adapt to future scenarios. Equation 4.56 is the maximum allowable power flow for existing lines. Equation 4.57 is the maximum allowable power flow for candidate lines. Equation 4.58 is the core-trajectory update equation for generation investment. Equation 4.59 represents adaptation from the core-trajectory to different scenarios. Equation 4.60 and 4.61 signifies that the decision variables involved must be non-negative. Equation 4.64 is the allowable power a generator is allowed to dispatch in an operating condition.

4.2 Procedure

There are several steps involved in planning using adaptation. The 5 steps involved in transmission planning are listed below:

1) Selection of transmission candidates
2) Scenario generation
3) Scenario reduction
4) Design of transmission using adaptation
5) Validation of design

4.2.1 Selection of transmission candidates

The first step in the procedure is selection of transmission candidate lines. Transmission line are selected to ensure that the planning problem is feasible for the planning horizon.

4.2.2 Scenario generation

After several global uncertainties have been identified, then each global uncertainty is therefore assigned with a number of realizations, for example natural gas can be a global uncertainty with realization of high medium and low natural gas price trajectory. This research does not model local uncertainties.

4.2.3 Scenario reduction

After generating multiple scenarios the next step performed is scenario reduction. Scenario reduction can help reduce the computation complexity of a problem. By reducing the number of scenarios by clustering similar scenarios together, the number of variables and constraints are directly reduced, therefore making the problem computationally tractable. Techniques such as SP uses approaches such as simultaneous backward reduction, fast forward selection and scenario tree construction [52]. An approach is developed for scenario reduction in this dissertation. This approach computes the optimal investment for all considered scenarios and then tries to find similarities between the optimal solutions and then clusters the scenarios based on this similarity.

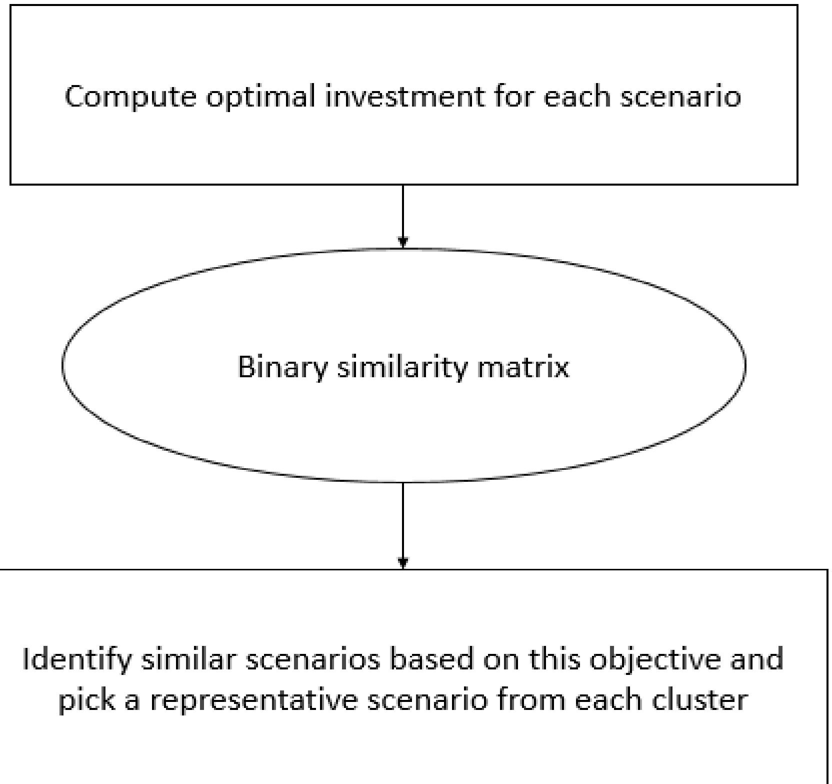

Figure 13: Figure for scenario reduction approach

TISI (Transmission Investment Similarity Index)

The similarity index is used to measure how similar optimal plans for each scenario are. We use the phi-co-efficient correlation index. The range for this index is [-1,1]. The phi-coefficient correlation index is described in [53]. We describe its use in this dissertation as follows.

When solving the optimal deterministic solution for each scenario, a vector for each scenario's solution is constructed indicating the transmission investments identified in that solution. (We delete any variables corresponding to transmission candidates that were never invested in any scenario. This provides that the vector of all scenario solutions is of minimal dimension and also so that the similarity index is reflects only essential information.

There are four possible combinations when comparing each element in two vectors. The variables M1, M2, M3, and M4 are the number of elements in both vector that correspond to the combinations described in the Table 4 below.

Table 4: Relationship between variables

	Variable 1	Variable 2
M1	0	0
M2	0	1
M3	1	0
M4	1	1

$$Phi_{index} = \frac{M_1 * M_4 - M_2 * M_3}{\sqrt{(M_4 + M_3)(M_4 + M_2)(M_3 + M_1)(M_1 + M_2)}} \quad (4.50)$$

GISI (Generation Investment Similarity Index)

This index measures the similarity between the generation investments of two scenarios. The closer the value to one the stronger the similarities between scenarios. After the optimal co-optimized solution is solved for all scenarios, the generation investment solution is separated for all scenarios and stored in a vector, the GISI computes the similarity between the vectors using their distance information.

$$d_{i,j} = \sqrt{\sum_{n=1}^{N} (v_{i,n} - v_{j,n})^2}$$

$$GISI = 1 - \frac{d_{i,j}}{\max(d_{i,j})}$$

where,

$d_{i,j}$ is the distance between vector i and j

N is the number of elements in the vector

$v_{i,n}$ is the nth element of vector i

vi,j is the nth element of vector j

Hierarchical clustering

Hierarchical clustering analysis (HCA) is an approach in data mining[54]. The information in which HCA is presented is known as a dendrogram. The dendrogram displays a hierarchical relationship between data. The information is presented as bottom-up or top down clusters that have sub-clusters and the sub-clusters also have sub-clusters and keeps going in that fashion. The HCA clusters based on the distance or similarity between data. The similarity matrix for both transmission and generation investment is a symmetric matrix and it is clustered using HCA approach. The horizontal -axis of the dendrogram represents number of individual observations while the vertical-axis represents the similarity/distance information. In our analysis the horizontal-axis represent scenarios.

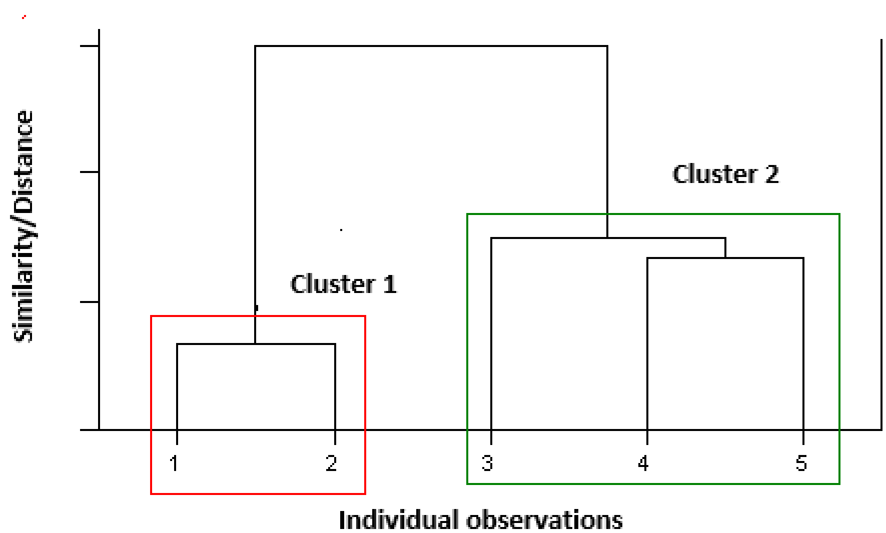

Figure 14: Dendrogram

4.2.4 Optimization of investment using adaptation

After scenario reduction, scenarios have been grouped into clusters. A representative scenario is selected from each cluster and explicitly modeled as a future in the adaptation formulation.

4.2.5 Validation

The main objective of the adaptation is to design a system that has good performance among a wide range of futures. In the validation phase, the adaptive design is compared to different deterministic designs. The deterministic designs selected are the optimal solution to representative scenario selected from each cluster. The idea of validation is to show there is benefit in considering uncertainty and that a design using adaptation is consistent in its performance across a wide range of scenarios.

4.3 Software Design Process

The data for the planning problem is stored in excel. A developed Matlab code reads the planning data as an input, after the planning data is read, another Matlab code generates the matrix required for optimization problem. The code is then run through the ECPE server at ISU.

Input data

-Fuel cost

-Demand

-Decision stages

-Generator data

-Scenarios

-Existing transmission lines

-Candidate transmission lines

"Xlsread" is the code used to read date from excel into Matlab. Aineq is the linear inequality constraints in a Matrix, while Aeq is equality constraints in a Matrix.The other vectors generated are

f=vector for objective values

lb = the lower bound vector for variables

ub = the upper bound vector for variables

The server in Fig.15 below has 94 GB in memory and 24 CPU's.

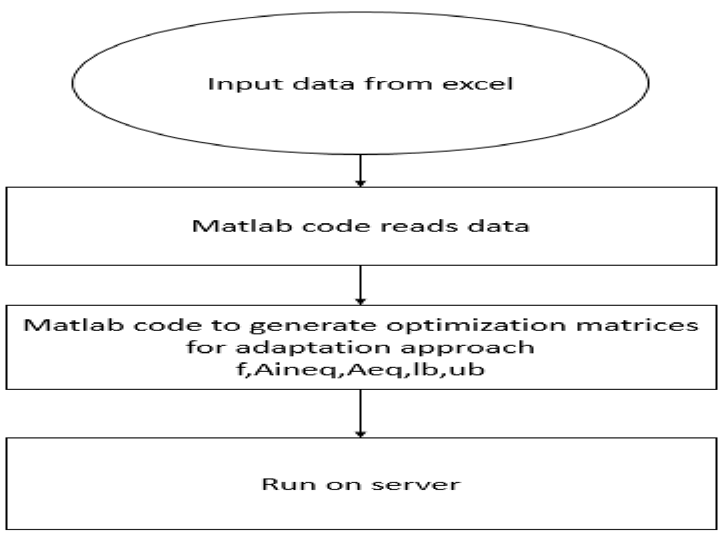

Figure 15: Software design process

CHAPTER 5. APPLICATION TO IEEE 24 BUS SYSTEM

5.1 Introduction

In this section adaptation is applied to transmission and co-optimization expansion planning and IEEE 24 Bus system is used for this case-study. In this co-optimization formulation, the transmission candidates decision variables are integer while the generation decision variables are continuous variables

5.2 Case-study

A Planning problem is for formulated and solved using the adaptation. The planning horizon is 20 years. In each year, 5 operating conditions are considered. A modified version of the IEEE 24 bus system is used for this case-study. Decisions are made before the 1st year, the 5th, 10th and 15th year. The first case-study is solely for transmission expansion and the second case-study illustrates co-optimization of both transmission and generation investment. There were 18 scenarios considered.

Table 5: Operating conditions

Operating condition	Load ratio	Hours
1	0.5115	438
2	0.6338	1751
3	0.6779	4381
4	0.824	2015
5	1	175

We consider three global uncertainties. These uncertainties, and the values they may take, are:

1) Natural gas price growth uncertainty

-High price: 3% per year

-Medium price: 2% per year

-Low price: 1% per year

2) Demand growth uncertainty

-High demand growth: 2.2% per year

-Medium demand growth, 1.5% per year

-Low demand growth, 1% per year

3) Carbon tax uncertainty

-Yes, $20/Mwh

-No

IEEE 24 BUS

The IEEE 24 Bus-system is used in this case-study. This system consists of 38 lines. The number of candidate circuits is also 38 lines. Figure 16 illustrates this system. Data for this system is provided in Appendix A. Table 6 identifies the 18 scenarios that are possible based on the attributes that each global uncertainty can take.

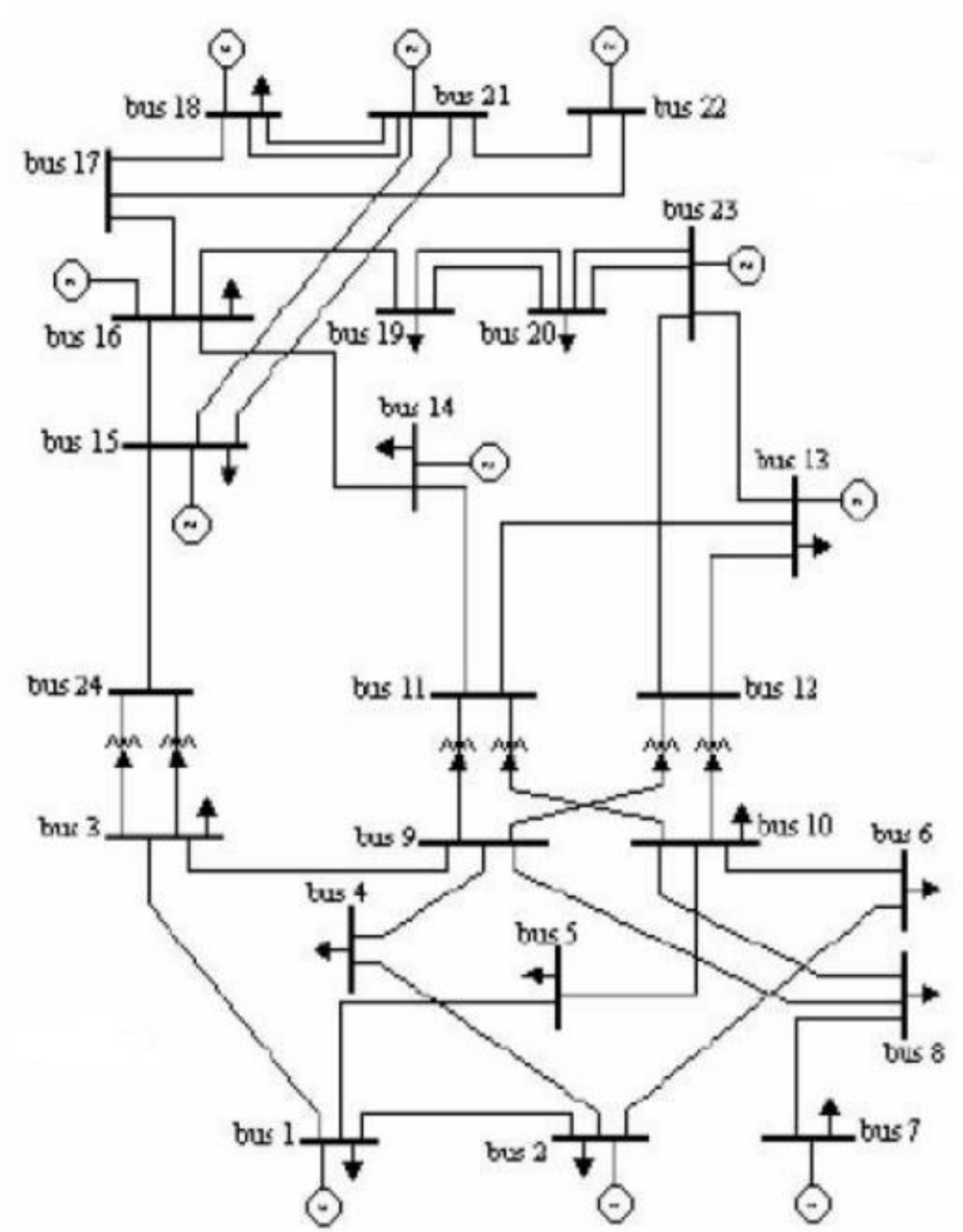

Figure 16: IEEE 24 Bus system

Table 6: Global scenarios

Scenarios	Natural gas price	Demand	Carbon tax
1	Low	Low	No
2	Low	Low	Yes
3	Medium	Low	No
4	Medium	Low	Yes
5	High	Low	No
6	High	Low	Yes
7	Low	Medium	No
8	Low	Medium	Yes
9	Medium	Medium	No
10	Medium	Medium	Yes
11	High	Medium	No
12	High	Medium	Yes
13	Low	High	No
14	Low	High	Yes
15	Medium	High	No
16	Medium	High	Yes
17	High	High	No
18	High	High	Yes

5.3.1 Scenario reduction for transmission planning

The scenario reduction technique is performed using hierarchical clustering technique using a dendrogram. The optimal solution is solved for each of the 18 different scenarios. A symmetric similarity matrix based on the phi-correlation co-efficient is computed. The hierarchical clustering technique is then used cluster the scenarios based on similarities with other scenarios.

Table 7: Optimal solutions for all 18 scenarios

			Scenarios																	
From	To	MW	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	2	175	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
1	3	175	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0
2	6	175	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0
3	9	175	0	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0	1	0
3	24	400	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
6	10	175	0	0	0	0	1	0	1	1	1	0	0	1	1	1	1	1	1	1
7	8	175	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
8	9	175	0	1	0	1	0	0	0	0	0	1	0	1	0	1	0	1	0	0
8	10	175	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	1	0	1
10	12	400	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	1	1
14	16	500	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
15	16	500	0	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0
15	21	500	0	0	0	0	1	0	0	0	0	0	1	0	1	1	0	1	1	0
15	21	500	0	0	1	0	1	0	0	0	1	0	1	0	0	0	1	0	0	1
15	24	500	0	0	1	0	1	0	0	0	1	0	1	0	0	0	0	0	1	0
16	17	500	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	19	500	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
17	18	500	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	22	500	0	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0

The table 7 above shows all the lines that were built in each scenario regardless of the time they were built.

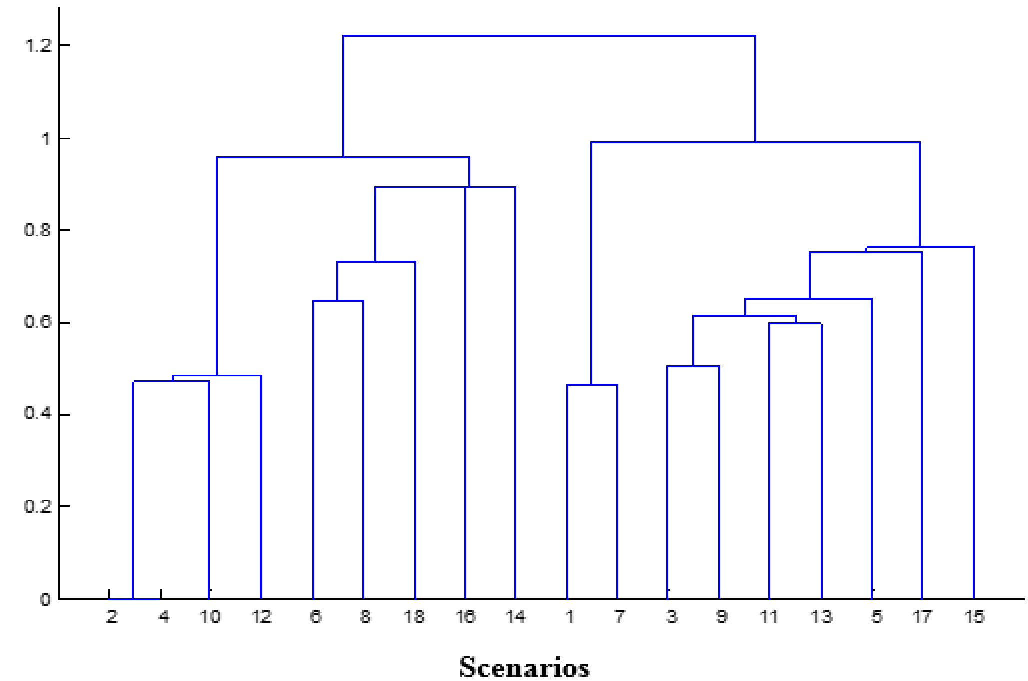

Figure 17: Clustering scenarios using a dendrogram

In the dendrogram of Figure 17, the y-axis depicts the strength of the clusters and the x-axis represents the scenarios involved, the lower the value of the y-axis, the stronger the clusters. The scenarios were clustered into six groups.

Cluster 1 {2, 4, 10, 12}

Cluster 2 {14, 16}

Cluster 3 {1, 7}

Cluster 4 {6, 8, 18}

Cluster 5 {3, 5, 9, 11, 13}

Cluster 6 {17, 15}

From cluster 1, scenario 2 and 4 have exactly the same solution, while scenario 10 and 12 built an extra line in additional to all the lines built in scenario 2 and 4(see Table 7). From the cluster 2, scenario 14 and 16 built the same lines. From cluster 3 Scenario 1 and 7 built the same lines aside from an additional one built by scenario 7. From cluster 4, all the lines built in scenario 6 and 8 except one built in scenario 6 are subsets of all the lines built in scenario 18. In the scenario clustering it can be seen that scenarios with carbon tax were never clustered with scenarios without carbon tax.

A selected scenario that well represents the cluster is selected and modelled explicitly in the mathematical model of adaptation. In this dissertation, there is no definite way for determining the number of clusters, however there is an approach which is used to determine whether a cluster is cluster-worthy and this is based on the following qualiliative assessment below.

-1.0 to -0.7 strong negative association.

-0.7 to -0.3 moderate negative association.

-0.3 to +0.3 little or no association.

+0.3 to +0.7 moderate positive association.

+0.7 to +1.0 strong positive association.

These ranges are very common in the statistical community. The minimum acceptable was the moderate positive association. Every cluster has a similarity matrix and the pair-wise correlation between two scenarios had to be either in the moderate positive association or strong positive association.

5.3.2 Results/Case-study

In the adaptation formulation, 6 representative of the selected scenarios from the pool of 18 scenarios are explicitly modelled in the formulation. However, in the validation process all the 18 scenarios are used. A design of β value of 0.25 is compared with optimal solution with 6 representative scenarios. There are 38 transmission candidates and this translates the problem to 836 binary variables.

Table 8 summarizes transmission investments made for the adaptation-based design with β=0.25. If a candidate was not invested, then it is not shown in Table 8. Thus, the left-hand column of Table 8 groups invested candidates and their capacity. Table 8 indicates that all transmission investments were made at the initial investment period (T=0), and none were made thereafter. This suggests that transmission of this system is initially insufficient to serve its load from the generation resources that it has. To check this, line flows were inspected for the initial peak load conditions, and it was found that all of the transmission candidates listed in Table 8 were at their limits.

Table 8: Core-trajectory for transmission investment (β=0.25)

Transmission Candidate (Bus i to Bus k)	Capacity	T=0	T=5	T=10	T=15
3 - 24	400MW	1	0	0	0
7 - 8	175MW	1	0	0	0
8 - 10	175MW	1	0	0	0
16 – 17	500MW	1	0	0	0
17 – 18	500MW	1	0	0	0

We validate these results using two different approaches. In both approaches, six deterministic designs (the optimal solutions for the six representative scenarios) are compared with an adaptive design obtained based on a value of β=0.25. The two validation approaches are described in what follows:

Validation approach 1: In this validation approach, the initial investment plan of the six deterministic designs and the one adaptive design are forced to adapt to each of the 18 original scenarios. This approach was deployed because it is typical in the SP literature to validate in this fashion.

Validation approach 2: In this validation approach, the *entire investment trajectory* of the six deterministic designs and the one adaptive design are forced to adapt to each of the 18 original scenarios. This approach was deployed because each of the solutions obtained (i.e., the six deterministic designs and the one adaptive design) actually specific trajectories through the entire planning horizon, and therefore testing of the various solutions necessarily means testing of the entire investment trajectory.

The results of validation approach 1 are described in this subsection. The results of validation approach 2 are described in the next subsection.

Figure 18 below provides the average total costs across all scenarios for the adaptation-based design (β=0.25) and different deterministic designs. Here, "total costs" refers to the sum of total investments (both the original investments as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. It can be seen from Fig 18 below that the adaptation based design is on average \$3.2 million lower than the best deterministic design.

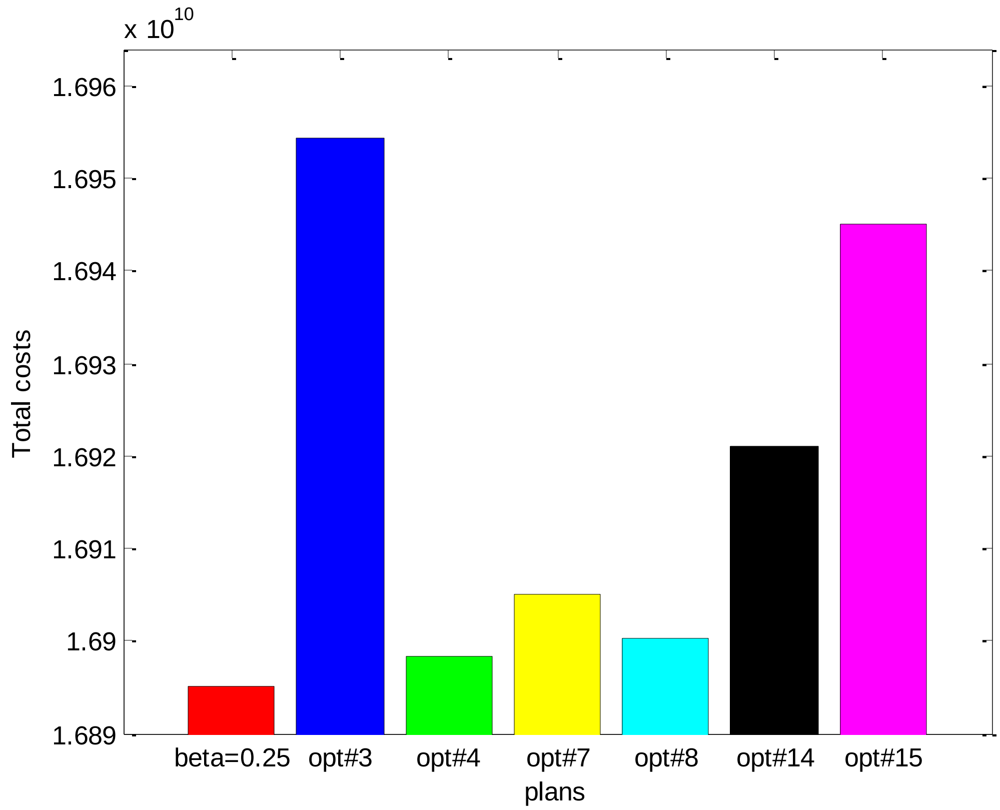

Figure 18: The average total costs across all scenarios for different deterministic and β designs

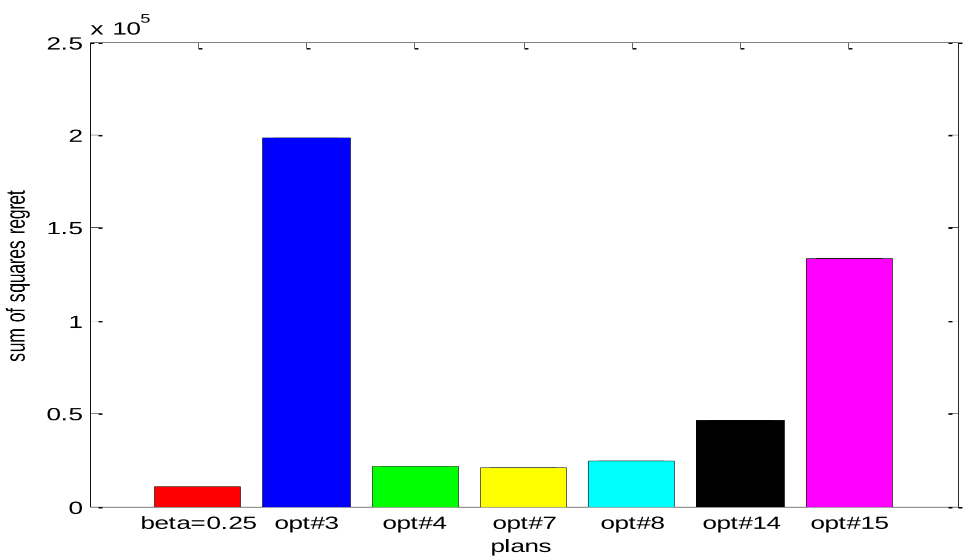

Figure 19: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 19 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the Figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

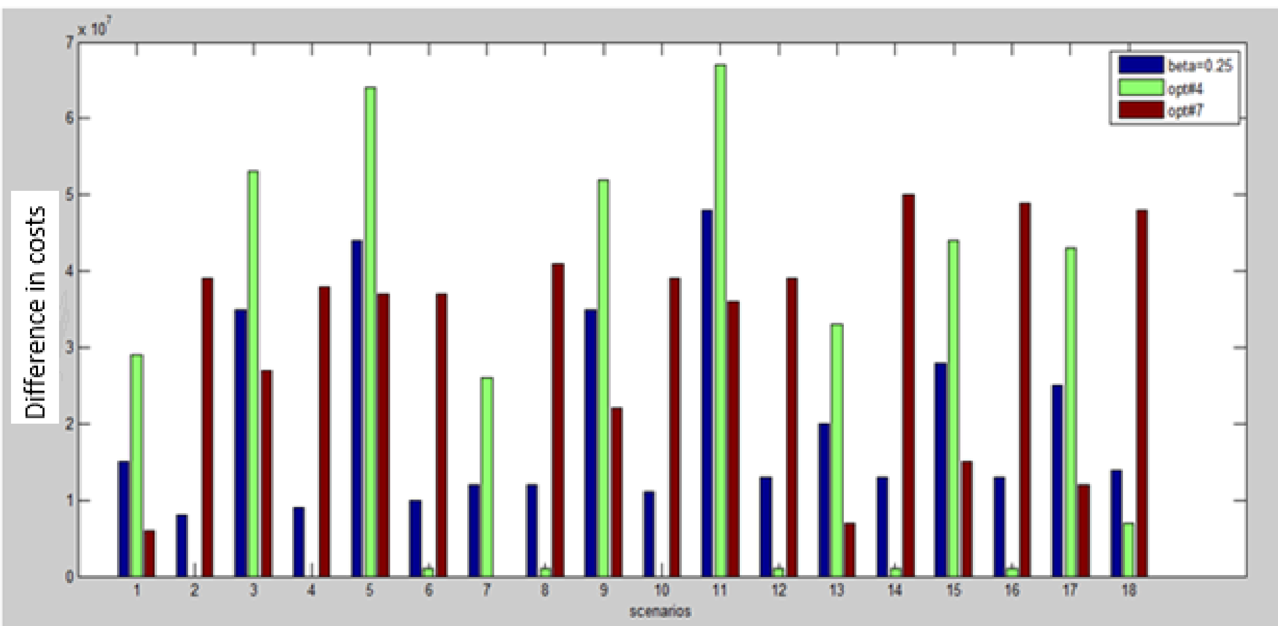

Figure 20: Comparison of the adaptation based design with the two deterministic designs in terms of total costs

It can be seen in the dendrogram in Fig.18 (and also in Table 6) that that scenarios with carbon tax were never clustered with scenarios without carbon tax. The two best deterministic designs are selected in terms of average total costs from scenarios with carbon tax and scenarios without; these are opt#4 and opt#7, as observed in Figure 18. These two designs, and the adaptation-based design, are then exposed to the 18 scenarios, and the adaptation costs are computed for each. Results are illustrated in Figure 20, where it is observed that the adaptation based design (is the blue bars) is never the highest in any of these scenarios. This shows consistency. It can also been seen in Figure 20 above that opt#4 performed well in scenarios with even numbers(i.e scenarios with carbon tax) and opt#7 performed well in scenarios with odd numbers (i.e scenarios without carbon tax). The value for opt#4 is zero for scenario 2 because from table 7 they built the same lines and in scenario 10 because they built same lines except from scenario 10 built an extra line, however if you expose scenario 4 solution to scenario 10 it builds that extra line.

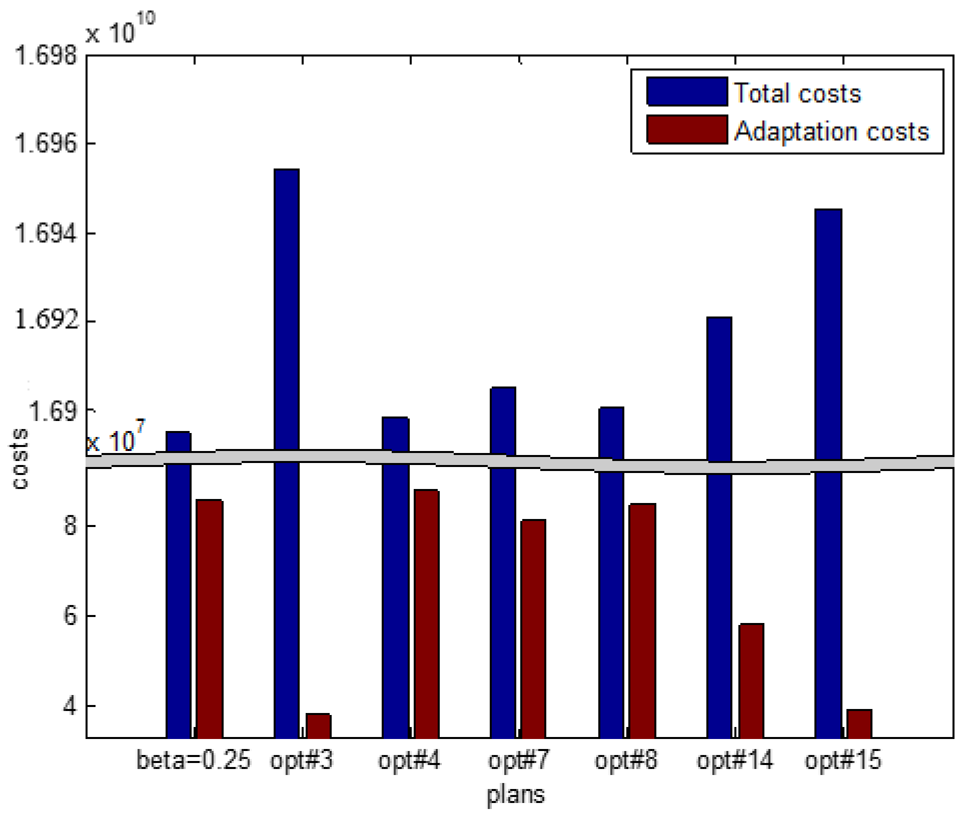

Figure 21: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 21 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. One idea of flexibility is to balance the total costs and adaptation costs. It can be seen in Fig. 21 above that although opt#3, opt#14 and opt#15 have lower adaptation costs they have very high total costs. Lower adaptation costs is a sign of a flexible plan, but it should be balanced with having a relatively low total costs.

Even though low β designs tends to have high adaptation costs, they tend to do well for decision-makers embracing "wait and see" philosophies, because they commit few resources initially but provide opportunity to adapt later, after the future scenarios have been revealed. The concept of "wait and see" differ in SP and adaptation. The difference here is that the "wait and see" option for SP is indeed a real "wait and see" option. In contrast, the "wait and see" option for adaptation is actually just the second of two decisions. The first decision is "what is the core investment?" and the second decisions is "what is the adaptation investment?". Selection of β is very influential in getting a good design. If β is not well chosen the design may be undesirable.

5.3.3 Validation using core trajectory

In this validation approach, the core-trajectory is used unlike in validation approach 1 where only the initial solution of the trajectory is used. This validation approach seeks to check the long-term adaptability of the core-trajectory. The six deterministic designs and the adaptive designs are forced to adapt to each of the 18 original scenarios. The total costs (i.e., investment costs + operations costs+re-investment costs) is computed as:

$$TC = Cost_{O\&M} + Cost_{CoreInvestment} + Cost_{Re\text{-}investment} \quad (5.1)$$

It can be seen in Fig. 22 below that the adaptation based design is lower than the deterministic design in terms of total costs.

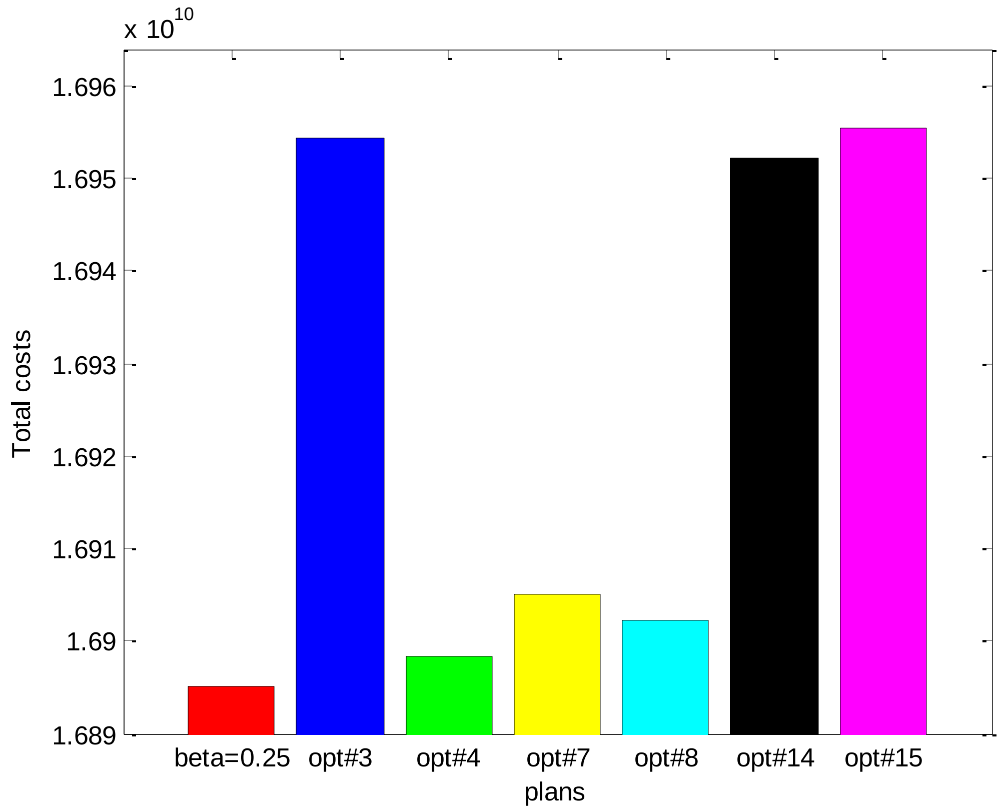

Figure 22: Average Total costs across all scenarios for different deterministic and β designs

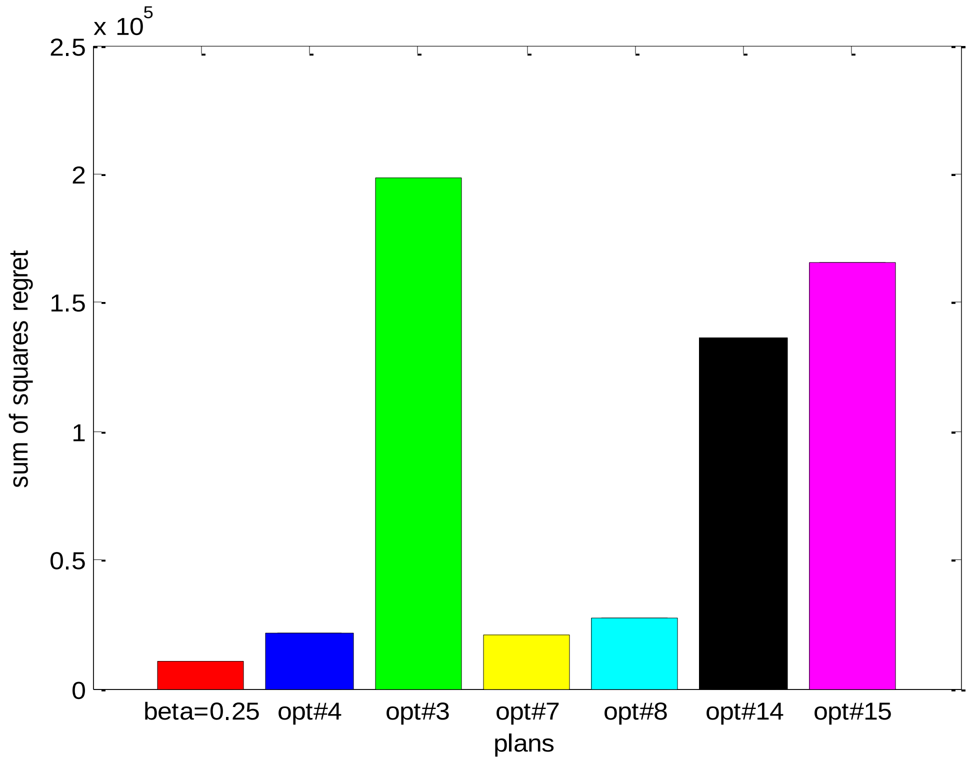

Figure 23: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 23 below. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

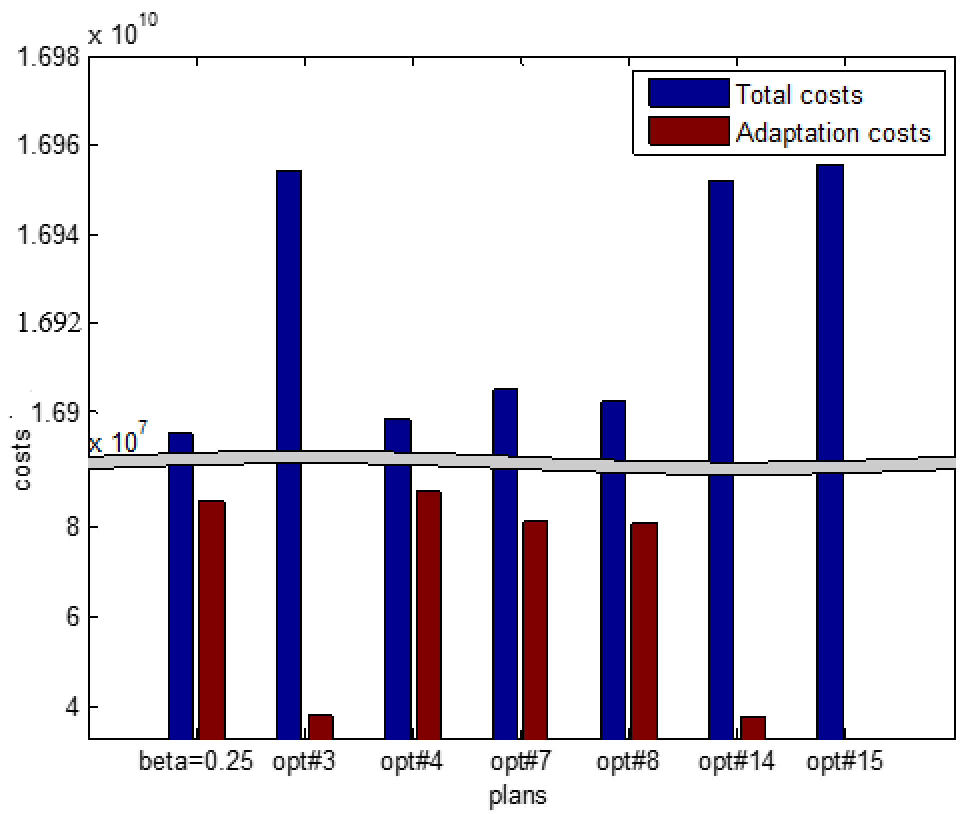

Figure 24: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 24 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen that in Fig. 24 above that Opt#15 has the lowest adaptation cost but a very high total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

5.4 Co-optimization Under Uncertainty Using Adaptation

In this section adaptation is applied to transmission and generation expansion planning using the IEEE 24 bus system. In this formulation the transmission candidates are model as integer while the generation decision variables are continuous.

A planning problem is for formulated and solved using adaptation. The planning horizon is 20 years. In each year, 5 operating conditions are considered (see Table 5). A modified version of the IEEE 24 bus system is used for this case-study. Decision are made before the 1st year, the 5th, 10th and the 15th years both for generation and transmission decisions. The base load used was 4000MW.

Global uncertainties, the attributes they can take, and the possible scenarios are the same as given for the adaptation done for a transmission expansion planning problem, and are given in Section 5.2

5.4.1 Scenario clustering for co-optimization

Scenario clustering for co-optimization is quite challenging because two decision variables are involved. An optimal solution for a scenario may have similar transmission investment solution but very different generation investment solution. An index is proposed that combines both similarity indices for both generation and transmission investment.

Generation and transmission investment similarity index

The generation investment similarity index (GISI) and the transmission investment similarity index (TISI) measure the similarity between the generation investments and transmission investments, respectively, of two scenarios. The closer the generation (for GISI) and the closer the transmission (for TISI) values from two investment solutions are to one another, the stronger the similarities between the two investment solutions. After the optimal co-optimized solution is solved for all scenarios, the generation and transmission investment solutions for all scenarios are stored in a vector and their similarity index is computed

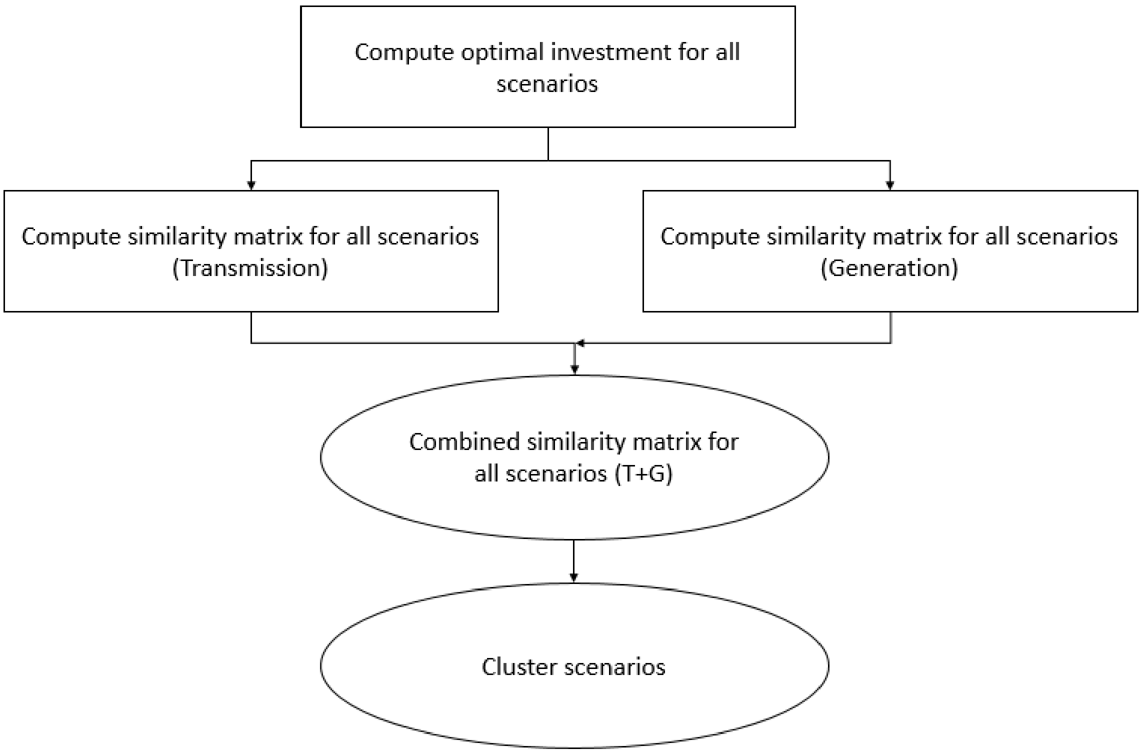

Figure 25: Scenario clustering approach for co-optimization

The combined index is the sum of squares of both G+T similarity index, since the transmission index(TISI) ranges from -1 to 1, the index is normalized to go from 0 to 1.

Table 9: Optimal solutions for all 18 scenarios (Transmission)

			Scenarios																	
From	To	MW	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	2	175	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	3	175	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1
1	5	175	0	0	0	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1
3	9	175	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0
3	24	400	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1
5	10	175	0	0	0	1	1	1	0	0	0	1	0	1	1	1	1	1	1	1
7	8	175	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
8	9	175	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1
8	10	175	0	1	0	1	0	0	0	1	0	1	0	0	0	1	0	0	0	0
15	16	500	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
16	17	500	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	19	500	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
17	18	500	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	22	500	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1

The table above shows all the lines that were built in each scenario regardless of the time they were built.

Table 10: Optimal solutions for scenarios 1-9 (Generation)

		Scenarios								
Bus	Gen Type	1 MW	2 MW	3 MW	4 MW	5 MW	6 MW	7 MW	8 MW	9 MW
1	Nuc	382.8	324.5	396.29	597.9	554.05	687.4	380.8	340.3	384.5
2	NG	0	0	0	0	0	0	0	0	0
7	Coal	0	0	0	0	0	0	0	0	0
13	NG	0	0	0	0	0	0	0	0	0
14	Wind	0	0	0	0	0	0	0	0	0
15	NG	866.5	1104.3	656.36	0	432.98	0	929.5	1048.2	1324.5
16	NG	0	0	0	0	0	0	0	0	0
18	Nuc	0	0	0	442.7	0	442.5	0	0	0
21	Coal	0	0	0	0	0	0	0	0	0
22	Nuc	70.9	102.7	217.83	373.1	404.17	380.1	180.7	223.4	177
23	Coal	0	0	0	0	0	0	0	0	0

The table above shows all the cumulative capacity built at each bus

Table 11: Optimal solutions for scenarios 10-18 (Generation)

		Scenarios								
Bus	Gen Type	10 MW	11 MW	12 MW	13 MW	14 MW	15 MW	16 MW	17 MW	18 MW
1	Nuc	620.8	588.9	674.3	643.6881	609.9309	629.8669	737.1	722.9	697.8
2	NG	0	0	0	0	0	0	0	0	0
7	Coal	0	0	0	0	0	0	0	0	0
13	NG	0	0	0	0	0	0	0	0	0
14	Wind	0	0	0	62.2091	82.6425	36.5837	24.7	21.5	0
15	NG	0	754.5	0	580.6342	555.5943	583.241	0	766.7	0
16	NG	0	0	0	0	0	0	0	0	0
18	Nuc	521.6	20.8	70.3	187.5304	259.6476	231.9153	562.8	129.2	175.9
21	Coal	0	0	0	0	0	0	0	0	0
22	Nuc	365.6	444.4	922	424.1999	371.2303	419.0348	369.4	431.3	928.4
23	Coal	0	0	0	0	0	0	0	0	0

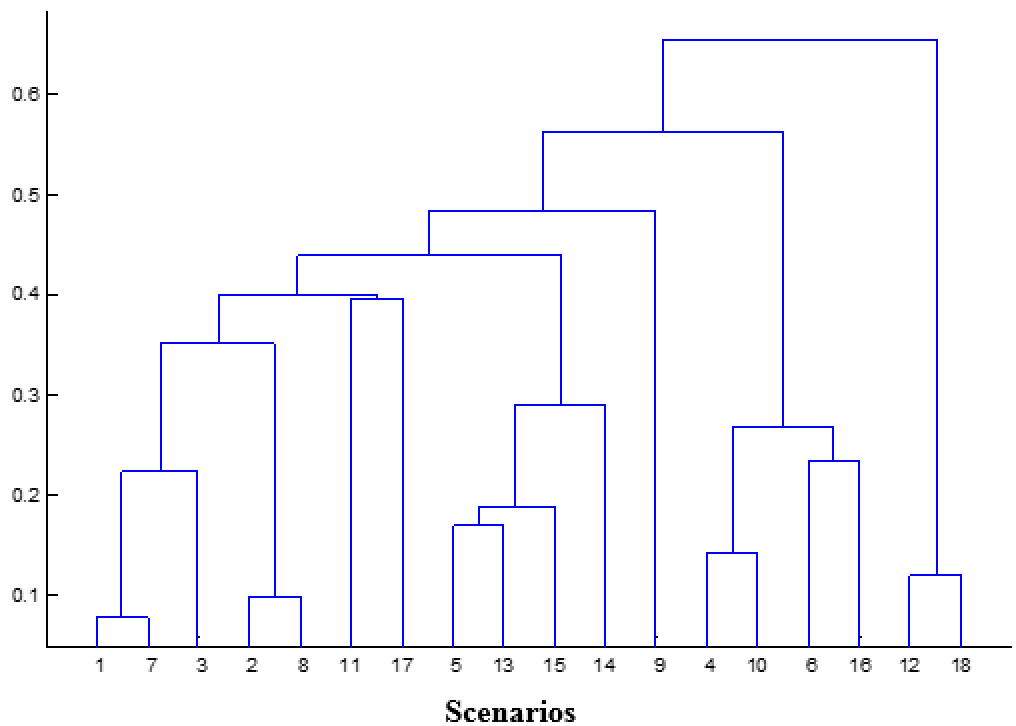

Figure 26: Dendrogram for scenario clustering for co-optimization

In the dendrogram in Figure 26, the y-axis depicts the strength of the clusters and the x-axis represents the scenarios involved, the lower the value of the y-axis, the stronger the clusters. The scenarios were clustered into seven groups.

Cluster 1 {1,3,7}

Cluster 2 {2,8}

Cluster 3 {11,17}

Cluster 4 {5,13,14,15}

Cluster 5 {4,6,10,16}

Cluster 6 {12,18}

Cluster 7 {9}

In cluster 1, for the transmission solution, scenario 1, 3 and 7 all built the same lines (see Table 9). In cluster 2, for the transmission solution, scenario 2 and 8 all built the same lines. In cluster 3, for the transmission solution, scenario 11 and 17 all built the same lines except that scenario 11 built line (16-19) and scenario 17 built line (7-8). In cluster 6, for the transmission solution, scenario 12 and 18 built the same lines except that scenario 18 built an extra line (15-16).

In cluster 1, for the generation solution, scenarios 1, 3 and 7 all built generation at the same location. The capacity of nuclear generation built at bus 1 is very similar (see Table 10).

In cluster 6, for the generation solution, scenarios 12 and 18 built generation at the same location. The capacity of nuclear generation built at bus 1 and bus 22 for both scenarios are very similar in terms of capacity (see Table 11).

A selected scenario that well represents the cluster is selected and modelled explicitly in the mathematical model of adaptation.

5.4.2 Results/Validation

In this case-study, scenario reduction resulted in 7 clusters. Of these, 1 contained only one scenario sometimes called an outlier. Clusters with just one scenario are excluded from further consideration because they will have less significant impact and because their inclusion significantly increases the computational requirements of the problem. For the each of the remaining clusters, one representative scenario was selected, so that the problem has 836 binary decision variables. The first problem solved is solely for transmission expansion, and the second problem solved is for co-optimization of both transmission and generation investment. There are 11 buses where new generation can be investment. The base load is 4000MW.

The table 12 is the core-trajectory investment for generation investment while Table 13 is the core-trajectory investment for transmission investment.

Table 12: Core-trajectory for generation investment (β_T=0.25, β_G=1)

Bus number	Generator type	T=0	T=5	T=10	T=15
1	Nuclear	435.4MW	0 MW	0 MW	0 MW
2	NG	0 MW	0 MW	0 MW	0 MW
7	Coal	0 MW	0 MW	0 MW	0 MW
13	NG	0 MW	0 MW	0 MW	0 MW
14	NG	0 MW	0 MW	0 MW	0 MW
15	Wind	824.2 MW	0 MW	0 MW	0 MW
16	NG	0 MW	0 MW	0 MW	0 MW
18	Nuclear	0 MW	0 MW	16MW	0 MW
21	Coal	0 MW	0 MW	0 MW	0 MW
22	Nuclear	62.9MW	69.7 MW	262.8MW	0 MW
23	NG	0 MW	0 MW	0 MW	0 MW

Table 13: Core-trajectory for transmission investment (β_T=0.25, β_G=1)

Transmission Candidate	Capacity	T=0	T=5	T=10	T=15
1-2	175MW	1	0	0	0
1-5	175 MW	1	0	0	0
16-17	500 MW	1	0	0	0
17-18	500 MW	1	0	0	0

The new investment in nuclear generation at bus 1 triggers the investment of two candidate lines (i.e., 1-2, 1-5). Table 13 does not show all candidates lines but shows only candidates lines that were chosen in the planning horizon.

The adaptation was run again with a lower value of β_G; it was decreased from β_G=1 to β_G=0.5. Results for the lower value of β_G are provided in Table 14 and Table 15. Comparison between the results in Table 12-Table 13 and Table 14-Table 15 indicate that the core-trajectory with a lower β for generation investments builds less generation and transmission capacity. This is intuitively satisfying because (a) lowering β_G, for generation, encourages higher adaptation costs and therefore lower core investment costs. The lower investment costs for generation drives less investment cost for transmission.

Table 14: Core-trajectory for generation investment (β_T=0.25, β_G=0.5)

Bus number	Generator type	T=0	T=5	T=10	T=15
1	Nuclear	325.5 MW	0 MW	0 MW	0 MW
2	NG	0 MW	0 MW	0 MW	0 MW
7	Coal	0 MW	0 MW	0 MW	0 MW
13	NG	0 MW	0 MW	0 MW	0 MW
14	NG	0 MW	0 MW	0 MW	0 MW
15	Wind	859.8 MW	0 MW	0 MW	0 MW
16	NG	0 MW	0 MW	0 MW	0 MW
18	Nuclear	0 MW	0 MW	0 MW	0 MW
21	Coal	0 MW	0 MW	0 MW	0 MW
22	Nuclear	0 MW	0 MW	0 MW	0 MW
23	NG	0 MW	0 MW	0 MW	0 MW

Table 15: Core-trajectory for transmission investment (β_T=0.25, β_G=0.5)

Transmission Candidate	Capacity	T=0	T=5	T=10	T=15
1-2	175MW	1	0	0	0
16-17	500 MW	1	0	0	0
17-18	500 MW	1	0	0	0

We validate these results using two different approaches. In both approaches, seven deterministic designs (the optimal solutions for the seven representative scenarios) are compared with an adaptive design obtained based on a values of (β_T=0.25 and β_G=0.5) and (β_T=0.25 and β_G=1). The two validation approaches are described in section 5.3.2

It can be seen in the Fig. 27 below that both of the adaptation based designs have the lowest costs.

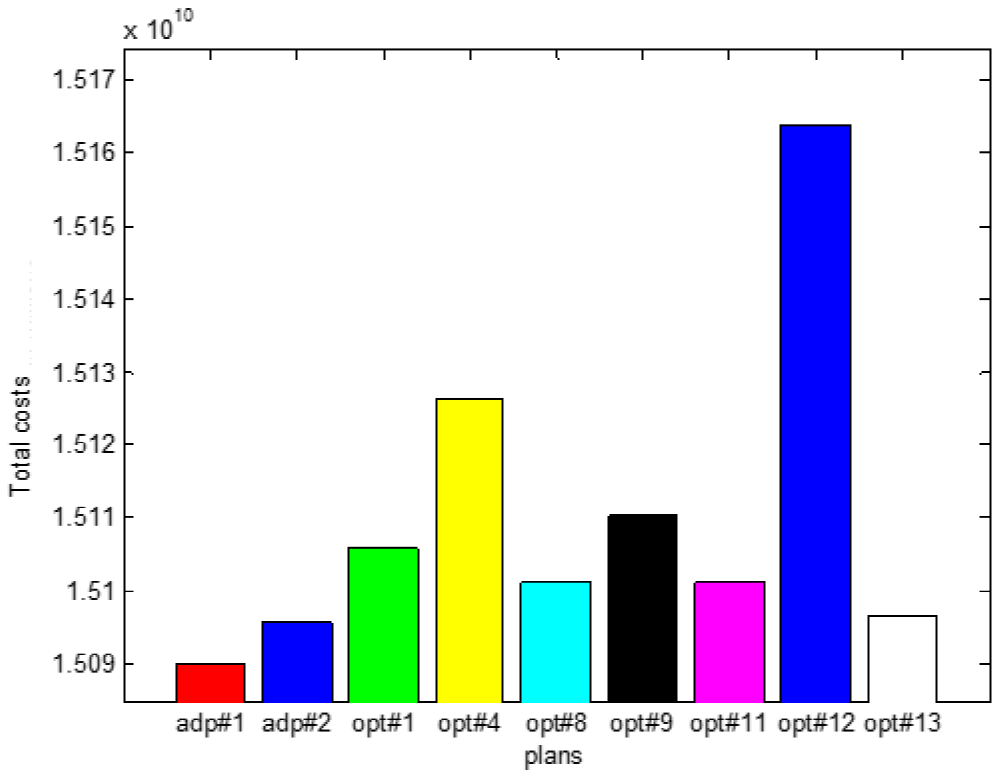

Figure 27: Average total costs across all scenarios for different deterministic and β designs

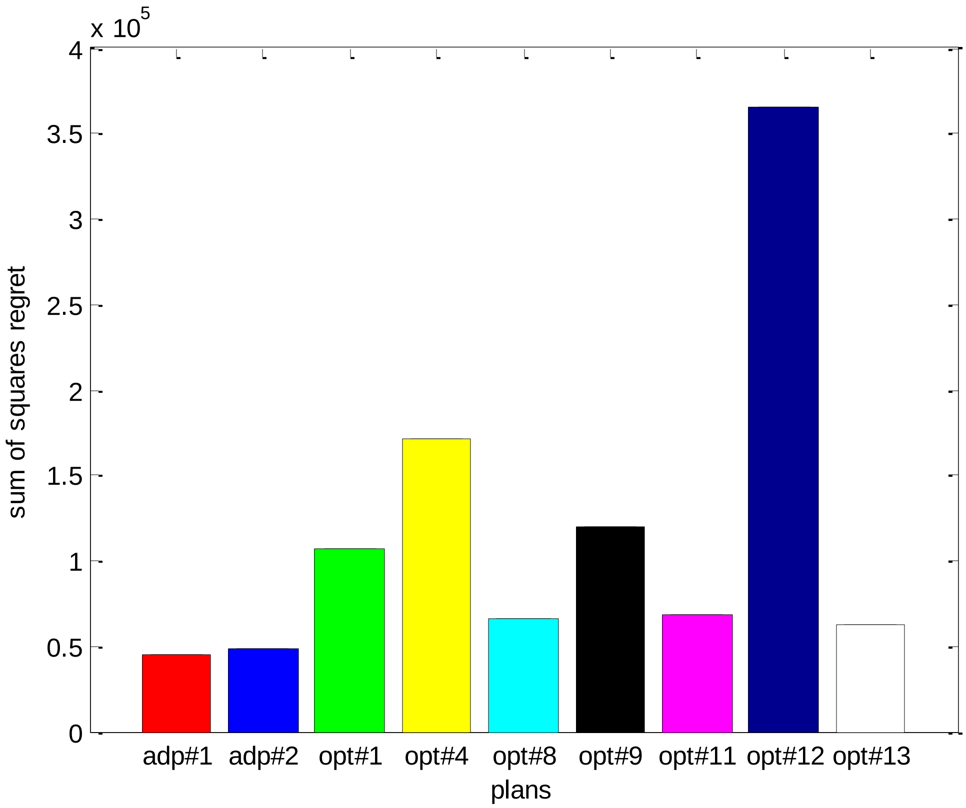

Figure 28: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 28 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

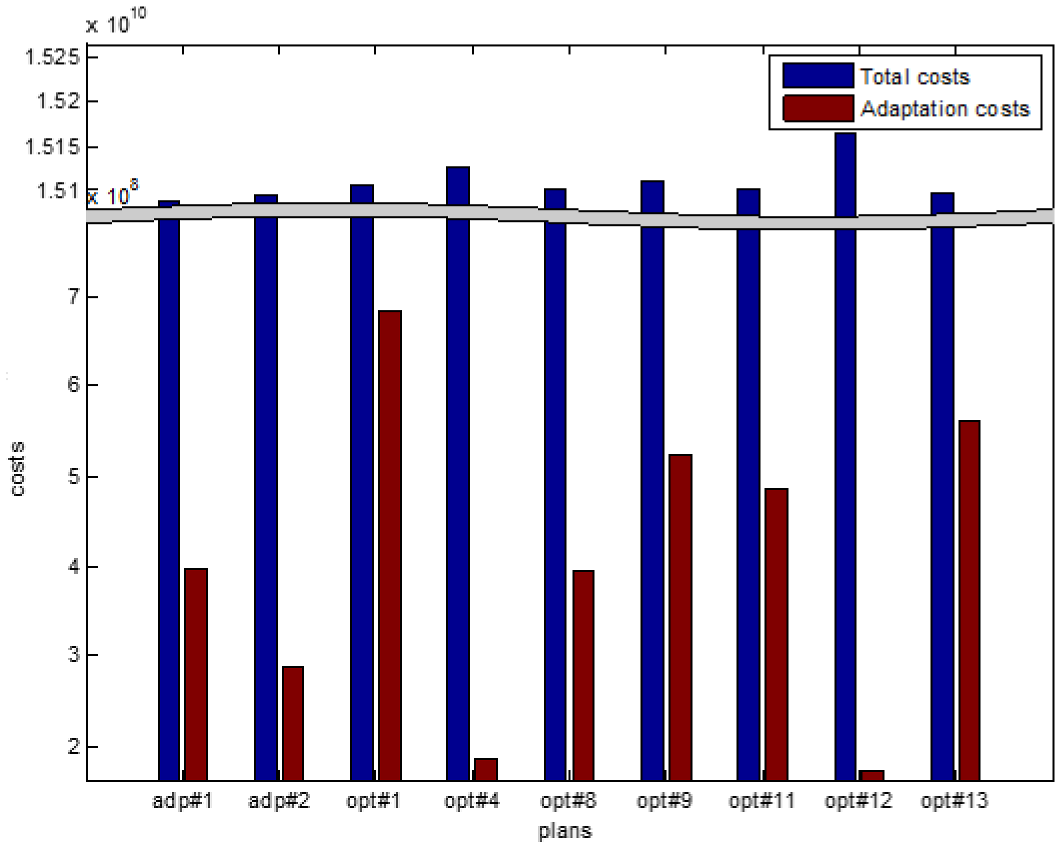

Figure 29: Average adaptation costs for all scenarios for different deterministic and β designs

The horizontal grey line in Fig. 29 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig.30 above that the Opt#12 has the lowest adaptation cost but has the highest total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

5.4.3 Validation using core trajectory

In validation we try to capture the value of including uncertainty. In order to validate our design. Deterministic design are compared with adaptive designs with different β. Seven deterministic design which are the optimal solutions for the seven representative scenarios. The seven deterministic designs and the adaptive designs are forced to adapt to 18 original scenarios. The total costs (i.e. investment costs + operations costs+re-investment costs) is computed. One of the characteristics of a flexible design is that it is consistent in performance across a wide range of scenarios. In this validation the core-trajectory is used unlike in the previous validation where only the initial solution of the trajectory is used. This type of validation seeks to check the long-term adaptability of the core-trajectory.

$$TC = Cost_{O\&M} + Cost_{CoreInvestment} + Cost_{Re\text{-}investment} \quad (5.2)$$

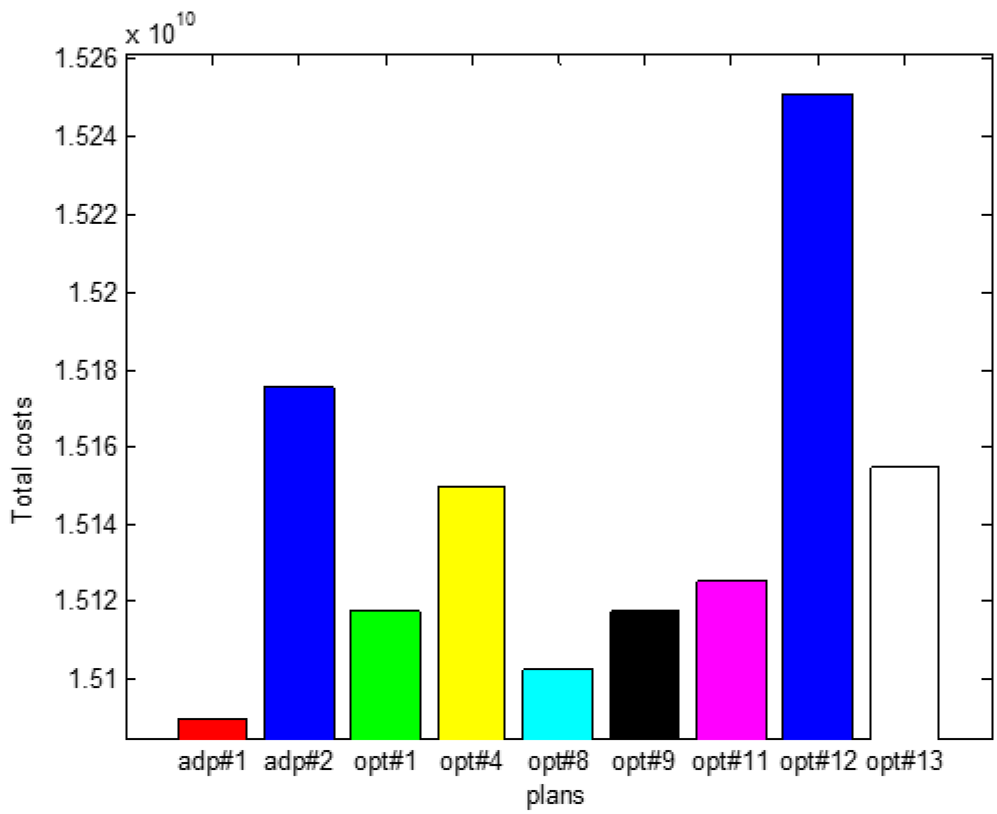

Figure 30: Average total costs across all scenarios for all scenarios for different deterministic and β designs

It can be seen in Fig. 30 above that the first adaptive design has the lowest total cost and the second adaptive design performs even worse than some deterministic designs, this has to do with selection of β. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs. The second design is more of a robust design than flexible design.

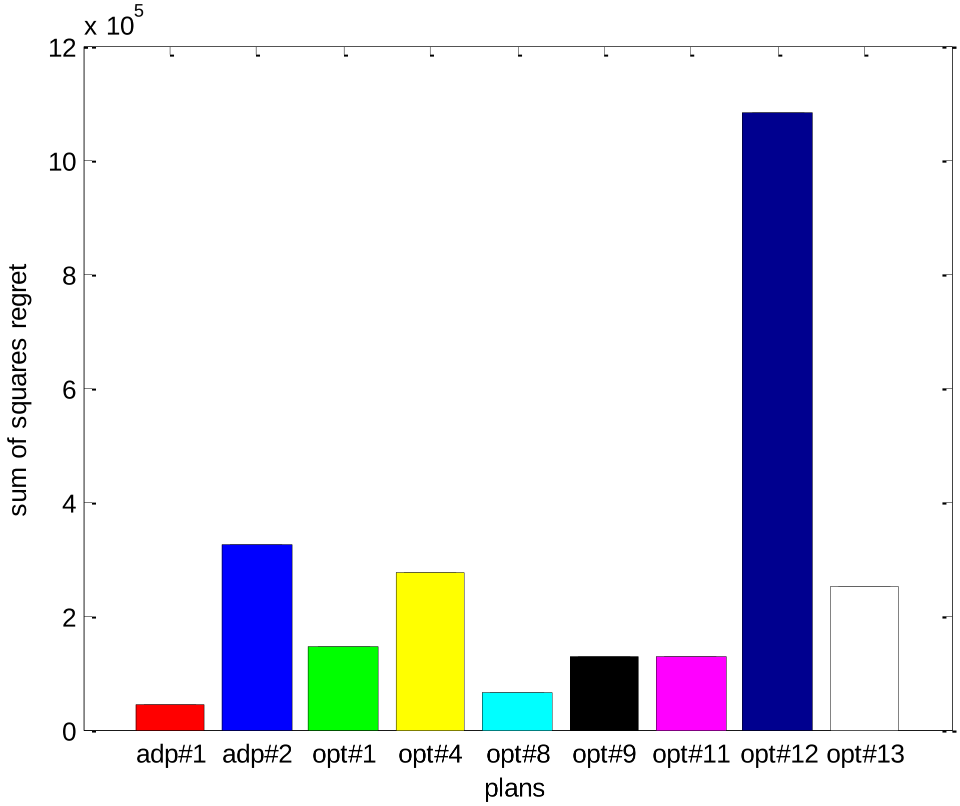

Figure 31: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 31 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

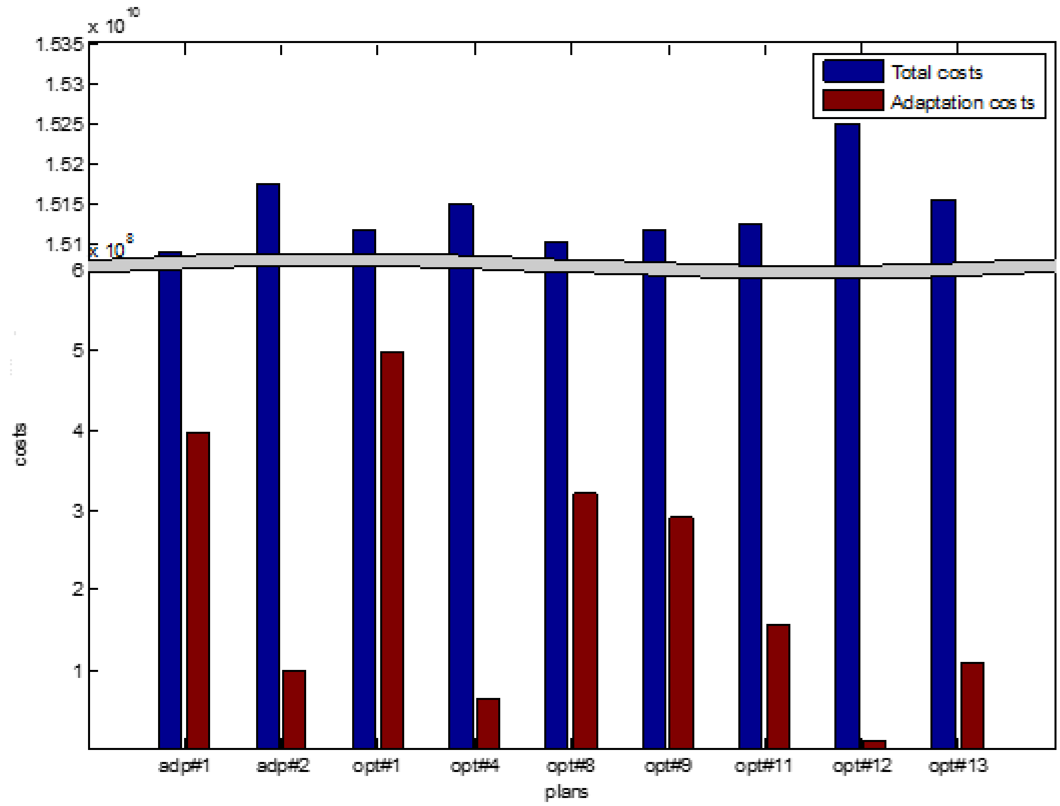

Figure 32: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 32 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig. 32 above that Opt#12 has the lowest adaptation cost but has the highest total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

CHAPTER 6. APPLICATION TO R2B DESIGN

This chapter applies adaptation to the design of "R2B" transmission under uncertainty. This chapter also describes the computational challenges faced and describes methods used to reduce the challenge. The first task done in this chapter is to design a backbone transmission to accommodate most of this new wind farms. James Slegers designed "R2B" under deterministic assumptions [55]. This dissertation extends the work and design "R2B" accounting for the issue of flexibility. After the initial design of "R2B" the future may warrant that that more wind farm is connected to the grid and the existing "R2B" transmission may need upgrades.

6.1 Characteristics Of A Good "R2B" Design

There are several good attributes of a R2B transmission. The electric power system planner will have to trade-off between these attributes.

Circuit miles

Many electric power utilities own thousands of circuit miles of transmission lines at different voltage levels. Transmission investment cost are a function of circuit miles (i.e. the longer the line, the more expensive the line is), also transmission lines with shorter circuit miles are easy to maintain than ones with longer circuit miles. In terms of reactive power, the longer the transmission the higher the possibility of insufficient reactive support for the transmission line. Therefore, it is necessary to minimize the number of circuit miles when considering transmission expansion.

Right –of –Way

The right-of-way of an electric transmission line is a lengthy limited region of land where construction, operation, maintenance and repair of transmission line equipment occurs. Securing right-of-way for transmission expansion is not an easy task. It is necessary to minimize the right-of-way of transmission lines due to the high costs associated with obtaining new right-of-way and also it will help minimize environmental impact introduced by power lines.

Visual Impact

Visual impact introduced by transmission lines is a very issues or concern for environmentalist and concerned citizens who appreciate scenic beauty. Transmission lines and other similar developments can adjust the aesthetics of nature. Therefore when planning transmission expansion, it is necessary to minimize visual impact introduced by transmission lines as much as possible. Placing transmission lines underground will solve the problem of visual impact; However, underground transmission is very expensive and could be as much as four times more expensive than overhead transmission lines. It is necessary to minimize the impact transmission lines towers and lines seen from residences, farms, roads, and recreational parks.

Reliability

Reliability has to do with the consistency of the quality of measurement. In electric power systems, reliability is the measure of the ability of a power system to adequately supply its electric energy demands. The reliability and availability of transmission must be very high. Reliability in electric system planning is one of the key factors that determine which expansion plans are to be invested. When choosing between transmission expansions alternatives, it is expedient to maximize reliability. Multiple paths increase system reliability because the failure of one line does not cause a system catastrophe.

Economics

Investment in transmission is a very delicate issue because investment is irreversible and transmission investment has a high sunk cost. Transmission investment cost consist of right-of-way costs, cost of material (i.e. conductor), cost of towers and poles, cost of labor. The cost of maintenance could also be quite expensive. The cost of transmission line is a function of how long in miles the line is and the MW capacity of the line. When comparing various transmission expansion alternatives, cost is a major factor. Transmission lines are very expensive and the cost vary according to the voltage level, MW capacity and length of line.

6.2 Uncertainty Modelling In “R2B” Design

When there are many possible future outcomes, uncertainty modelling is required in order to manage risk. The more uncertain an environment is, the more difficult it is to plan in the environment. Uncertainty has been a concern for decision makers and planners, especially when decisions made are irreversible. The following uncertainties are considered

a) Capacity Growth

b) Location of new capacity

6.3 Iowa Power System

The Iowa power system is used for this case-study. The Iowa power system consists of 338 existing lines ranging from 69 kV to 345 kV, 203 buses excluding neighboring states. Neighboring states such as Illinois, Minnesota, Missouri, Nebraska and Wisconsin are also modelled. James Slegers a former master’s student considered eight possible future wind resource areas considered in Iowa. The number of wind farms in each wind resource area varied from 6 wind farms to 18 wind farms. The figure below depicts how the wind-farms were clustered based on geographic locations; these locations were identified based on a systematic analysis that accounted for locational attributes including wind resource, proximity to existing transmission, and land unavailability (due to existing wind farms, national parks, municipalities).

The green boxes represent new wind-farms to be connected, while the red boxes represent possible substations to which the new wind farms can be connected.

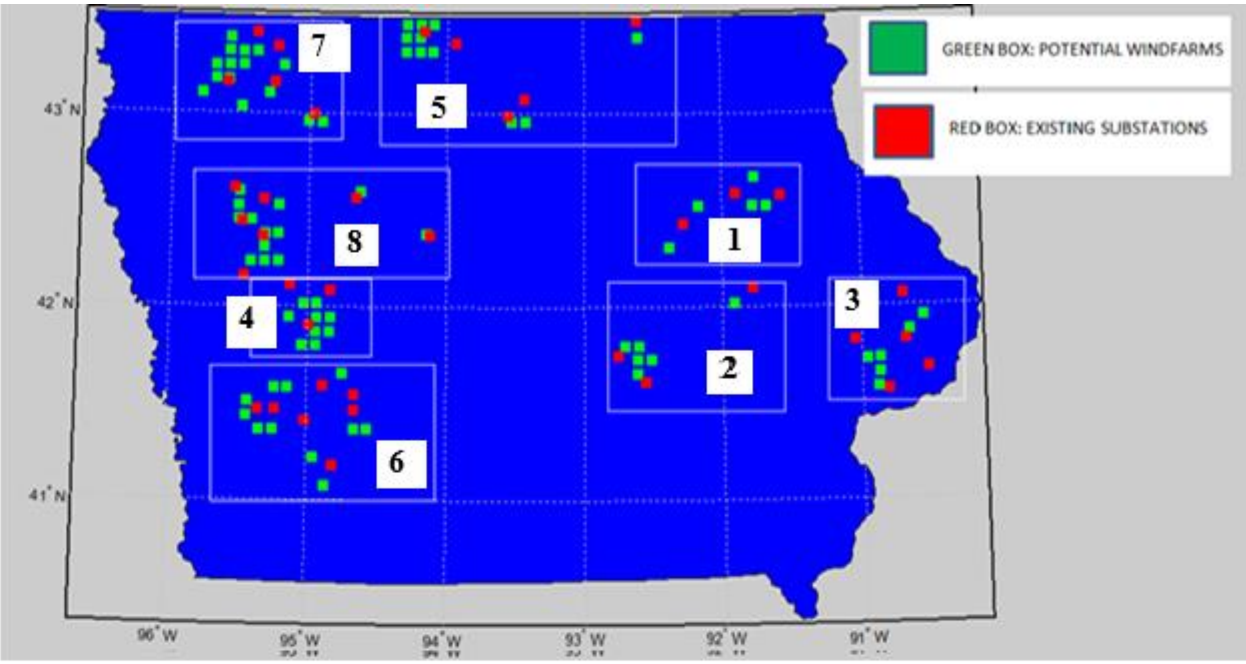

Figure 33: Location of wind-farms clusters on the map of Iowa

Table 16: Wind cluster and total capacity

Groups	Wind Farms	Capacity in MW
A	15	2963
B	11	2128
C	13	2572
D	9	1778
E	11	2163
F	5	982
G	6	1184
H	6	1185

6.4 Design Of A Backbone

Wind power is growing at a very fast rate. The need to connect multiple wind-farms to the main grid will be necessary. Wind resource rich areas tends to have low load ,hence high capacity backbone is needed to transfer most of this wind resources to high load areas. A backbone is designed for the state of Iowa. The optimization problem is formulated as a single period problem considering five operating conditions (i.e. combination of load and wind), the operating conditions can be seen in the Table 17 below. Each cluster of wind-farm is modelled as a large wind generator. Ninety-nine transmission candidates are considered and the St Claire curve in Fig. 35 is used to find the Surge impedance loading of transmission line candidates.

Table 17: Operating conditions

Operating condition	Load ratio	Hours	Wind output
1	0.5115	438	0.8202
2	0.6338	1751	0.6833
3	0.6779	4381	0.345
4	0.824	2015	0.1592
5	1	175	0.087

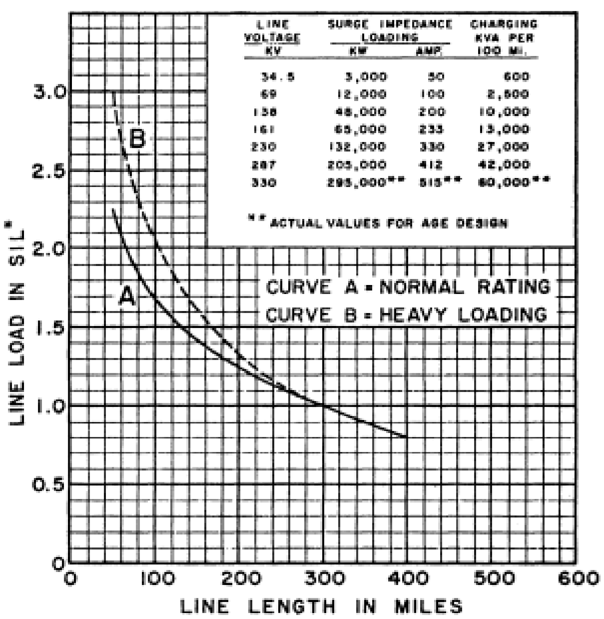

Figure 34: St-Claire curve [56]

The black lines in Fig. 36 below represent 765 kV lines.

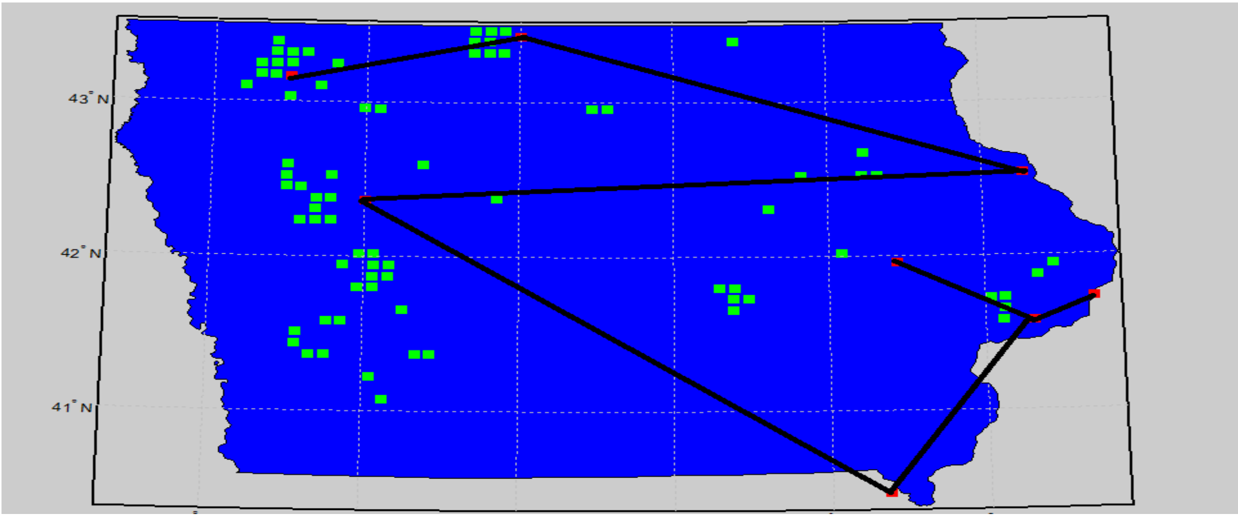

Figure 35: Backbone design for the state of Iowa

6.5 Case-Study/R2B Transmission

Possible future wind resource areas in Iowa were divided into 8 clusters in figure 33. The number of wind farms in a cluster varies from 6 wind farms to 18 wind farms. In this case-study, six of the clusters are used in this chapter. These designs are N-1 secure and all connected. The uncertainty addressed in this study is based on location and wind capacity growth uncertainty. We select different values of β to get different designs of R2B transmission, after selecting different values of β, we go through a validation process. Validation is performed for all six clusters. The planning is 10years and decisions are made at t=0 and t=5.

Selection transmission candidates for windfarms

In selection of transmission candidates, substations that have a lot of available transfer capability (ATC) had more transmission line candidates from wind farms connected to them. Wind farms that were considered very far from possible available substation based on longitude and latitude data were not given priority as transmission line candidate selection. The cost of transmission candidates is estimated based on the capacity and length of the line. Wind Clusters with more wind-farms had more transmission candidates.

Scenario generation

In scenario generation, we considered two kinds of uncertainty which are location and capacity uncertainty. Wind clusters with few wind farms we divided based on an individual wind farms, however wind clusters with a lot of wind farms were classified based on group of wind farms. For examples a wind cluster with 13 wind farms might be divided into 4 groups based on their proximity to each other. In the next stage of wind farm expansion, the location can either increase in wind capacity or remain constant. For example if a particular wind cluster is divided into 4 groups, in the next stage there will be a maximum of 16 scenarios (i.e. 2^4).

Contingency modelling

Contingency is modelled using the approach in this paper [44]. In this formulation the number of constraints and continuous variables is directly proportional to the number of contingencies considered, while the number of binary variables stays the same regardless of the number of constraints considered.

6.5.1 Results

Case-study #1 for windfarm group #1

The figure for wind cluster #1 can be located on the Iowa map in section 6.3

Table 18: Stage 1 wind-farm capacities

WF/MW	1	2	3	4	5
	197	197	191	199	197

Scenario generation

Since there are 5 buses, we consider that in the next stage, each bus can either increase by 150MW or not. This gives a maximum of (i.e. 2^5) 32 scenarios. In this case-study, 32 scenarios are used, we solve for the optimal investment for each scenario separately.

Scenario reduction

The scenario reduction technique is performed using hierarchical clustering technique using a dendrogram. The optimal solution is solved for all scenarios. The transmission similarity index is computed to measure the similarity between the transmission investments of two scenarios. The closer the value is to 1.0, the stronger the similarities are between the scenarios.

Table 19: Optimal solutions for scenarios (1-16)

			Scenarios															
From	To	MW	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	2	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	3	200	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
1	6	200	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1
1	6	200	1	0	1	1	0	1	1	1	1	0	1	0	1	1	1	1
1	6	400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	6	400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	8	200	0	1	0	0	1	0	0	0	0	1	0	1	0	1	1	1
1	8	200	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
2	3	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	3	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	4	200	1	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0
2	8	200	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	8	200	0	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1
3	6	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	6	200	1	0	1	1	1	1	0	0	0	1	1	1	1	0	0	0
4	8	200	1	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1
4	8	200	0	0	1	1	1	0	0	0	0	1	1	1	0	1	1	1
4	8	400	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
4	8	400	1	1	0	0	0	1	1	1	1	0	0	0	1	0	0	0
5	7	200	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1
5	7	200	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1
5	7	400	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0
5	7	400	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0

Table 20: Optimal solutions for scenarios (17-32)

			Scenarios															
From	To	MW	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	2	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	3	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	6	200	0	0	0	1	1	1	1	1	1	1	0	1	1	1	1	0
1	6	200	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0
1	6	400	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
1	6	400	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
1	8	200	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0
1	8	200	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	3	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
2	3	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	4	200	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0
2	8	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	8	200	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
3	6	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	6	200	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	1
4	8	200	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
4	8	200	1	1	1	1	1	0	0	0	1	0	1	1	1	0	0	1
4	8	400	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0
4	8	400	0	0	0	0	0	1	1	1	0	1	0	0	0	1	1	0
5	7	200	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
5	7	200	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
5	7	400	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
5	7	400	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0

Scenario reduction

The figure 36 below describes the dendrogram for scenario clustering.

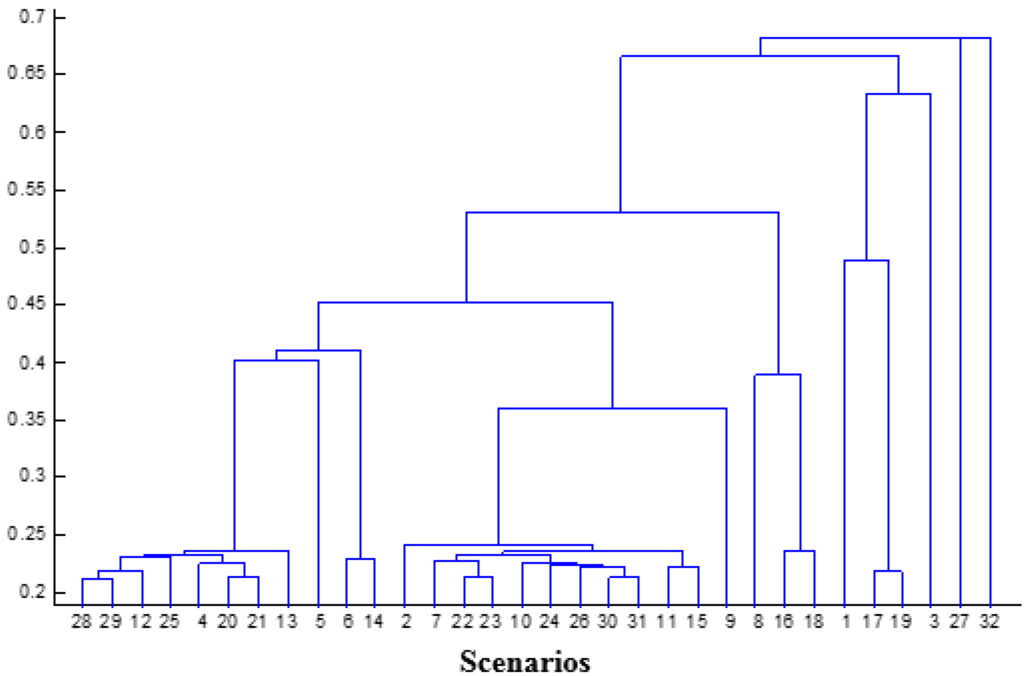

Figure 36: Dendrogram for scenario clustering for wind cluster #1

Group 1 {28, 29, 12, 25, 4, 20, 21, 13}

Group 2{ 5,6, 14}

Group 3 {2,7,22,23,24,26,30,30,31,11,15,9}

Group 4{8, 16,18}

Group 5 {1,7,19,3}

Group 6 {27}

A representative scenario is selected from each cluster and modeled explicitly in the adaptation formulation.

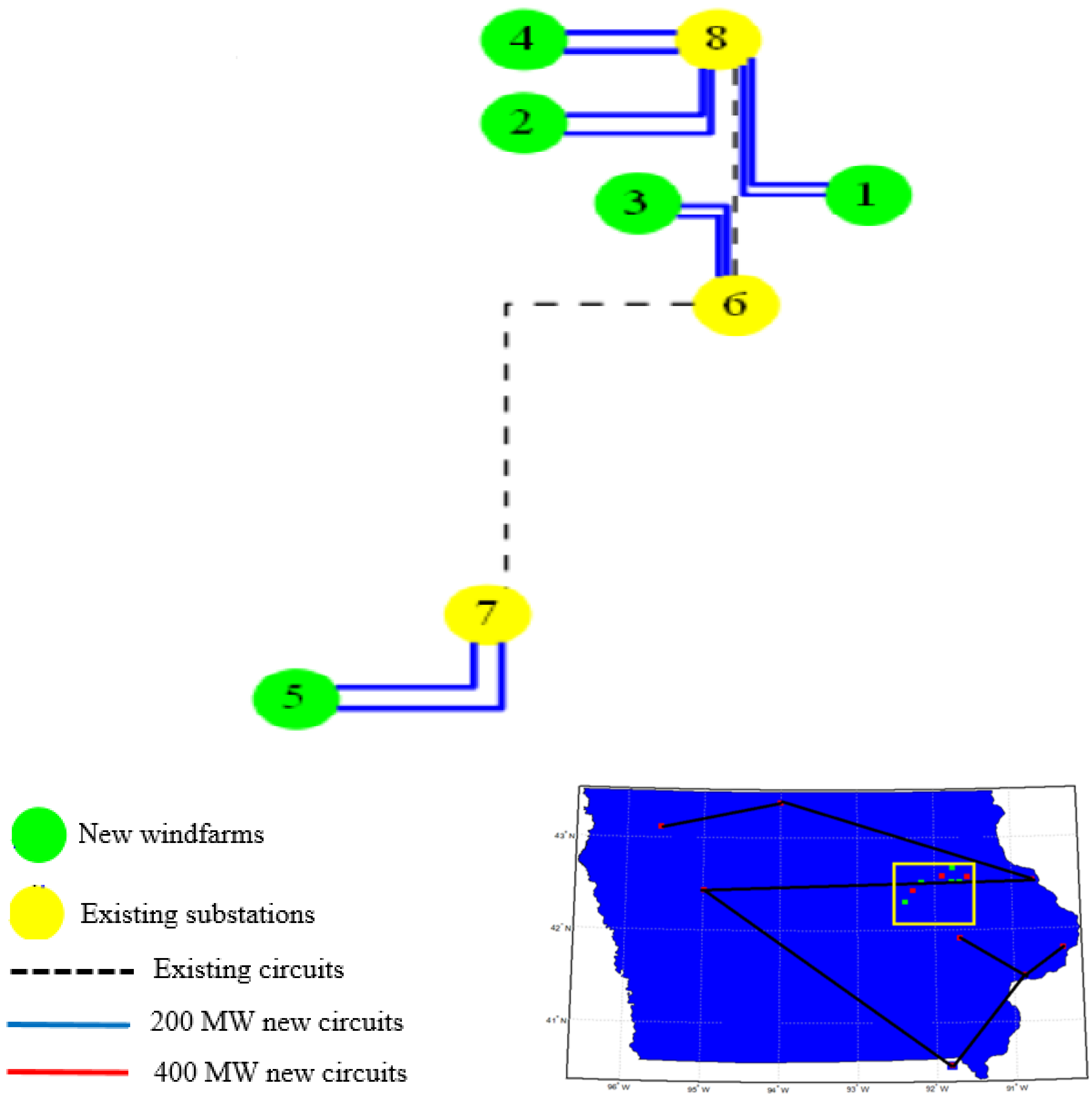

Figure 37: Design for β= 1(Core-trajectory)

All designs are N-1 secure and all the wind farms are connected.

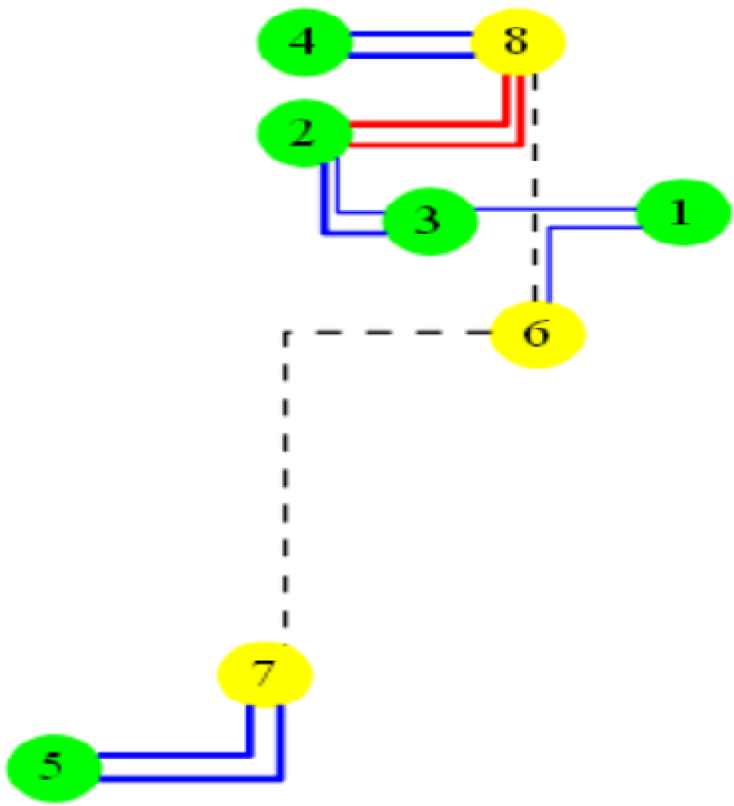

Figure 38: Design for β=0.25(Core-trajectory)

Validation

We validate these results using two different approaches. In both approaches, six deterministic designs (the optimal solutions for the six representative scenarios) are compared with an adaptive design obtained based values of β=0.25 and β=1. The two validation approaches are described in chapter 5.

The results of validation approach 1 are described first and the results for validation approach 2 are described next.

Approach #1

Figure 39 below provides the average total costs across all scenarios for the adaptation-based design (β=0.25 and β=1) and different deterministic designs. Here, "total costs" refers to the sum of total investments (both the original investments (i.e. initial investment) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. It can be seen in Fig.39 below that β=1 design performs best among all of the designs and opt#29 performs worst among all the designs.

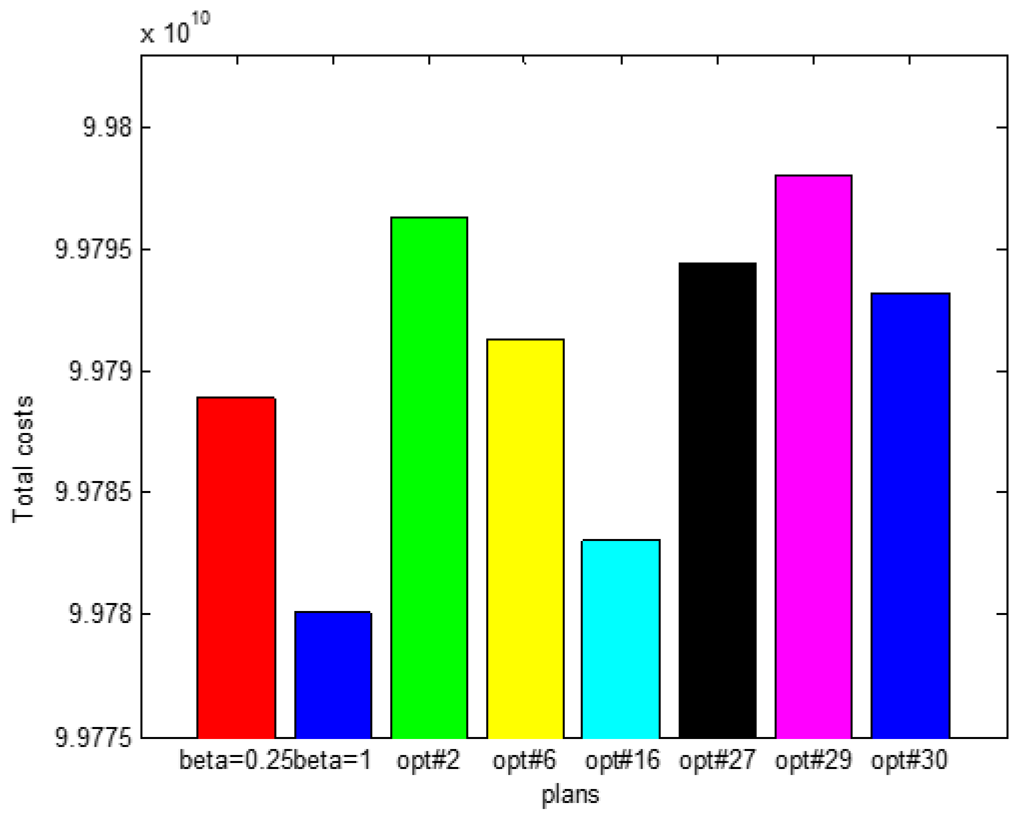

Figure 39: Average total costs across all scenarios for different deterministic and β designs

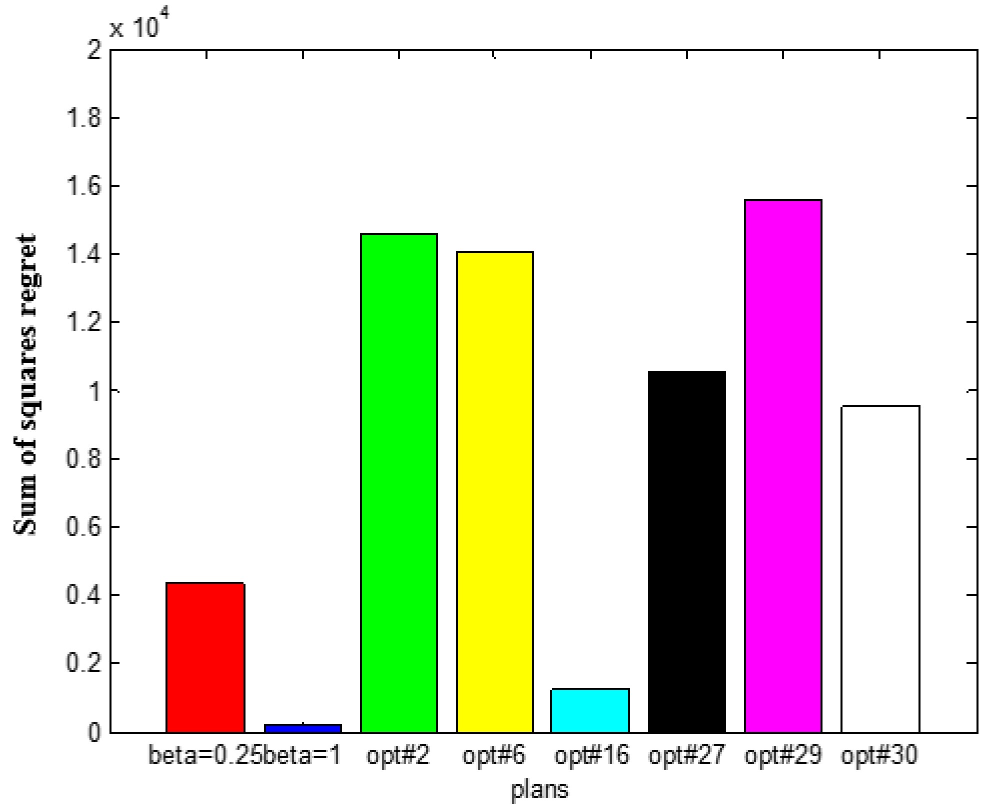

Figure 40: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig.40 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

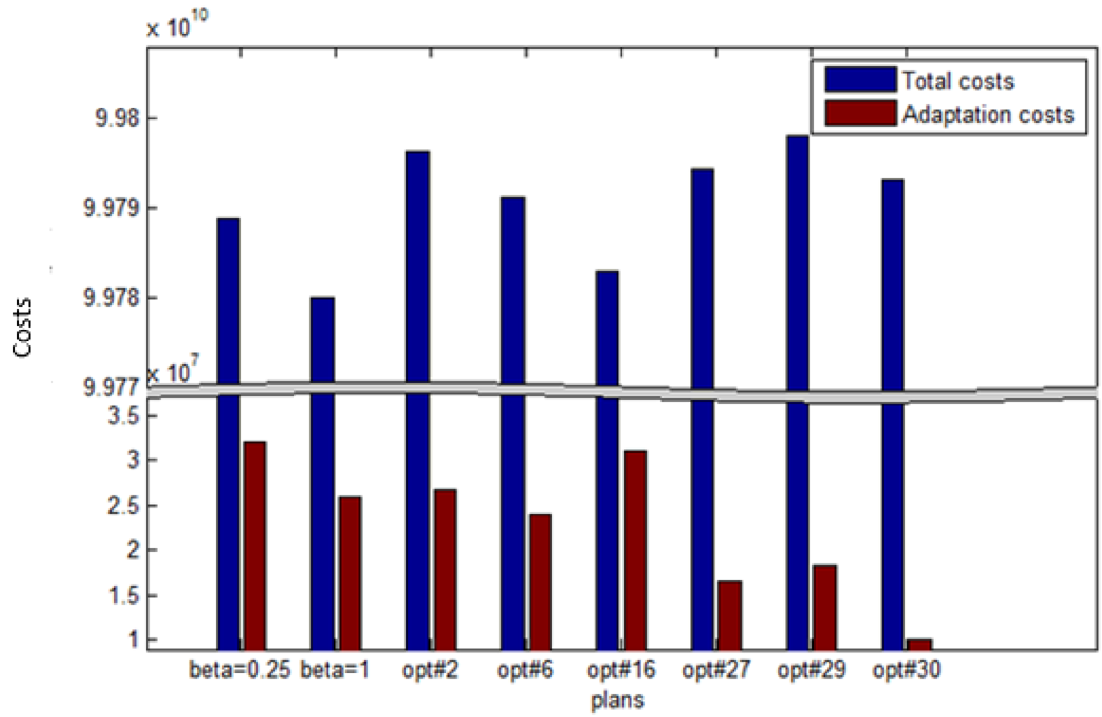

Figure 41: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 41 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen that in Fig. 41 above that although Opt#27 and Opt#29 have low adaptation cost but have a high total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs. It can seen in Fig. 41 above, that the design with lower β has a higher adaptation costs than the design with higher β.

Approach #2

Figure 42 below provides the average total costs across all scenarios for the adaptation-based design (β=0.25 and β=1) and different deterministic designs. Here, "total costs" refers to the sum of total investments (both the original investments (i.e. cost of core trajectory) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. It can be seen in Fig. 42 below that β=1 design performs best among all of the designs and opt#29 performs worst among all the designs.

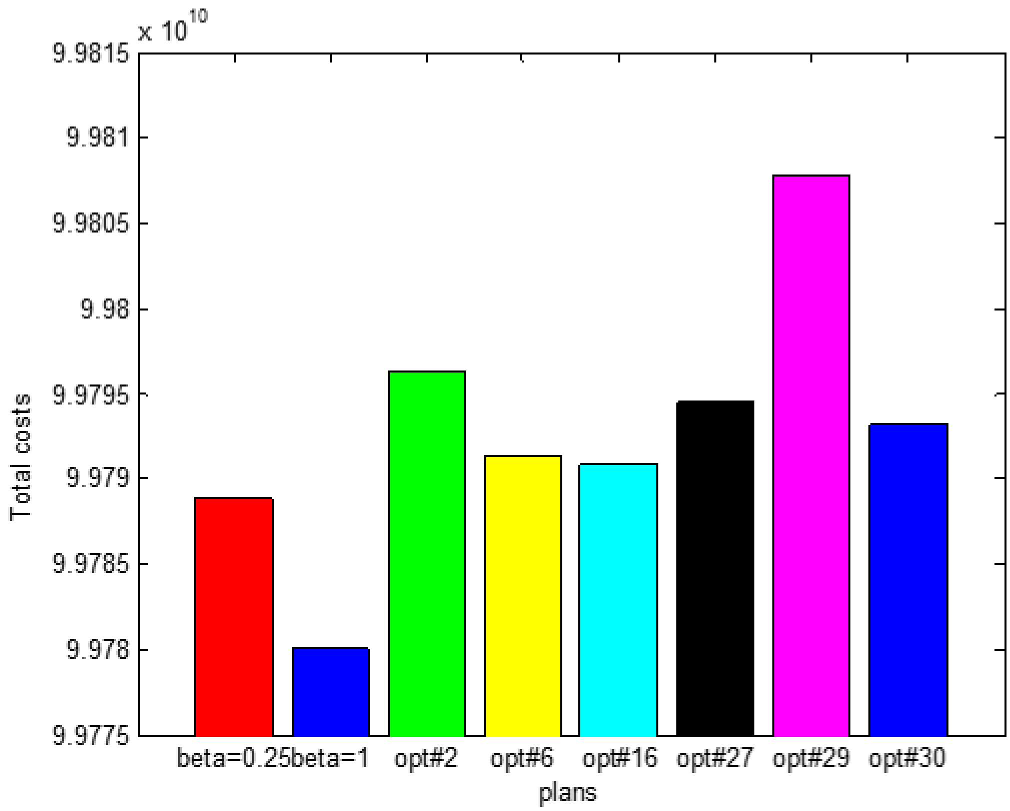

Figure 42: Average total costs across all scenarios for different deterministic and β designs

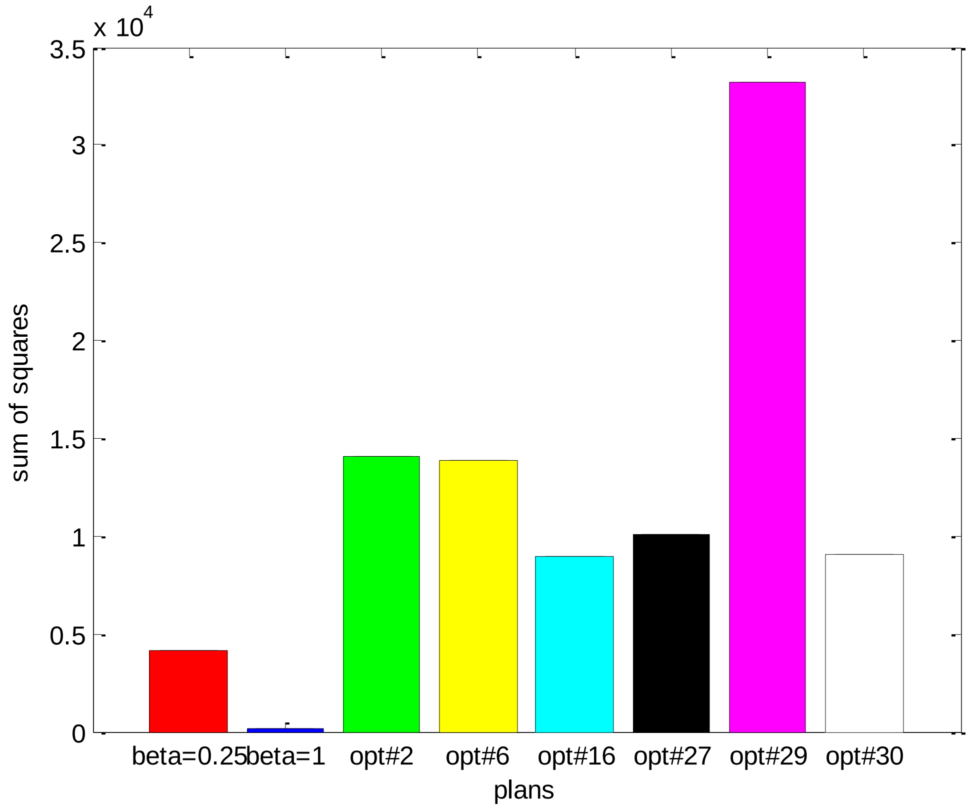

Figure 43: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 43 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario."

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

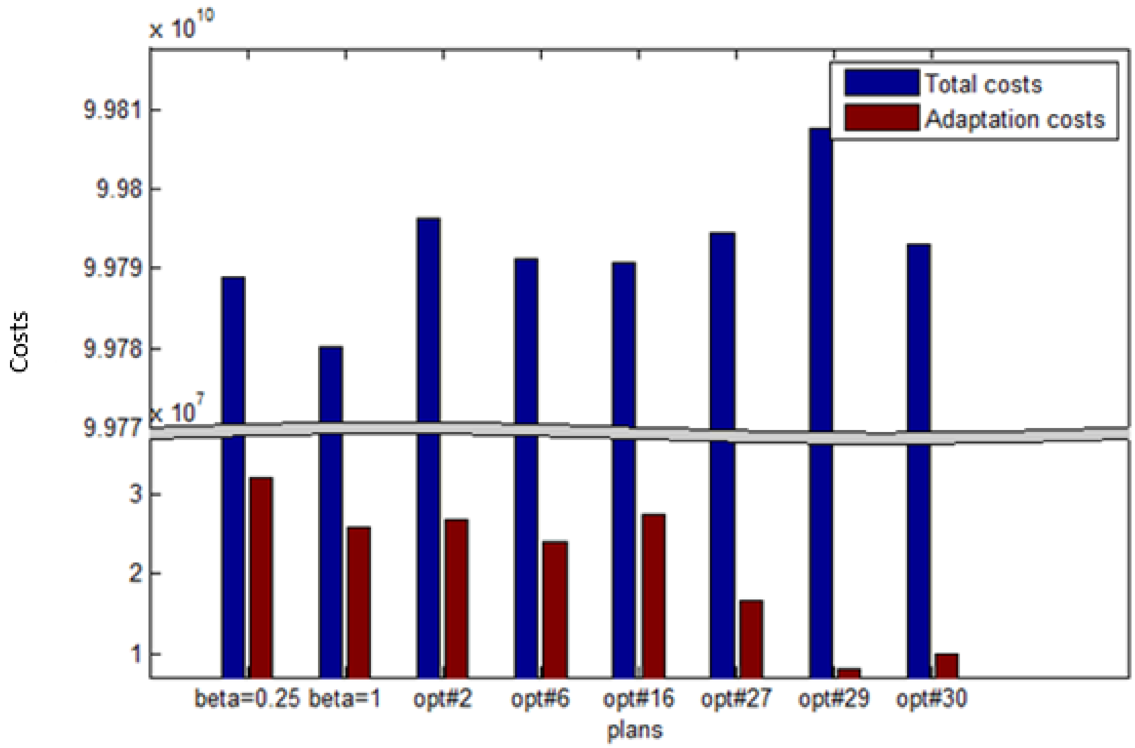

Figure 44: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 44 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig. 44 above that Opt # 29 has the lowest adaptation costs but has the highest total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

Case-study #2 for windfarm group #2

The figure for wind cluster #1 can be located on the Iowa map in section 6.3.

Table 21: Stage 1 wind-farm capacities

WF/MW	1	2	3	4	5	6
	197	200	200	198	193	196

Scenario generation

We divide the number of buses based on longitude and latitude data in to four areas {1} {2,3} {4,5} {6}.Therefore the 6 buses are divided four areas, we consider that each bus in an area can either increase by 150MW or not in the next stage. This gives a maximum of 16 (i.e 2^4) scenarios. In this case-study, 16 scenarios are used, we solve for the optimal investment for each scenario separately.

Scenario reduction

The scenario reduction technique is performed using hierarchical clustering technique using a dendrogram. The optimal solution is solved for all scenarios. The transmission similarity index is computed to measures the similarity between the transmission investments of two scenarios. The closer the value to one the stronger the similarities between scenarios.

Table 22: Optimal solutions for all scenarios

			Scenarios															
From	**To**	**MW**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	7	200	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	7	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	7	400	0	0	0	0	1	1	0	1	0	1	0	1	0	1	1	1
2	3	200	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0
2	3	200	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
2	4	200	0	0	0	1	0	1	0	0	0	0	1	0	1	0	0	1
2	6	200	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
2	9	200	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
2	9	200	1	1	1	1	1	0	1	1	1	1	0	1	0	1	1	1
2	9	400	0	0	0	1	0	1	0	0	1	0	1	0	1	0	0	0
2	9	400	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
3	4	200	1	1	1	0	1	0	1	1	1	1	0	1	1	1	1	1
3	4	200	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
3	5	200	0	0	0	1	0	1	0	0	1	0	0	0	1	1	0	0
3	8	200	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1
3	8	200	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
3	9	200	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
4	5	200	1	1	1	0	1	0	1	1	0	1	1	1	0	0	1	0

Table 22 continued

4	6	200	0	0	0	1	0	1	1	0	1	0	1	0	1	1	1	1
4	6	200	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
4	8	200	1	1	1	1	1	1	0	1	1	1	1	0	1	1	0	1
4	8	200	1	1	1	0	1	0	0	0	0	1	0	0	0	0	0	1
4	8	400	0	0	0	0	0	0	1	1	0	0	0	1	0	0	1	0
4	9	200	0	0	0	0	0	0	1	1	1	0	1	0	0	1	1	0
4	9	400	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
5	8	200	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
5	8	200	0	0	0	1	0	1	1	1	1	0	1	1	0	1	1	1
5	8	400	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
5	8	400	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
6	8	200	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1
6	8	200	1	0	0	1	1	1	0	1	0	1	1	1	0	1	1	0
6	8	400	0	1	1	0	0	0	1	0	1	1	0	0	1	1	1	1
6	8	400	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	1

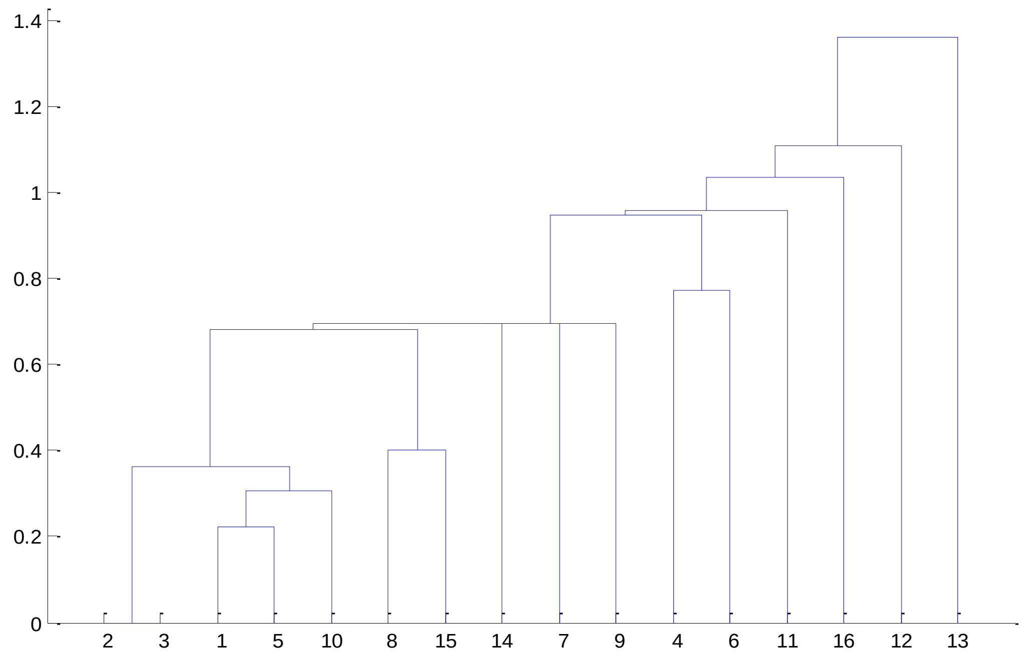

Figure 45: Dendrogram for scenario clustering for wind cluster #2

Cluster 1 { 1,2,3,5,10}

Group 2 {8,15,14,7,9}

Groip 3{ 4,6,11}

Group 4 {12}

Group 5 {16}

Group 6 {13}

A representative scenario is selected from each cluster and modeled explicitly in the adaptation formulation.

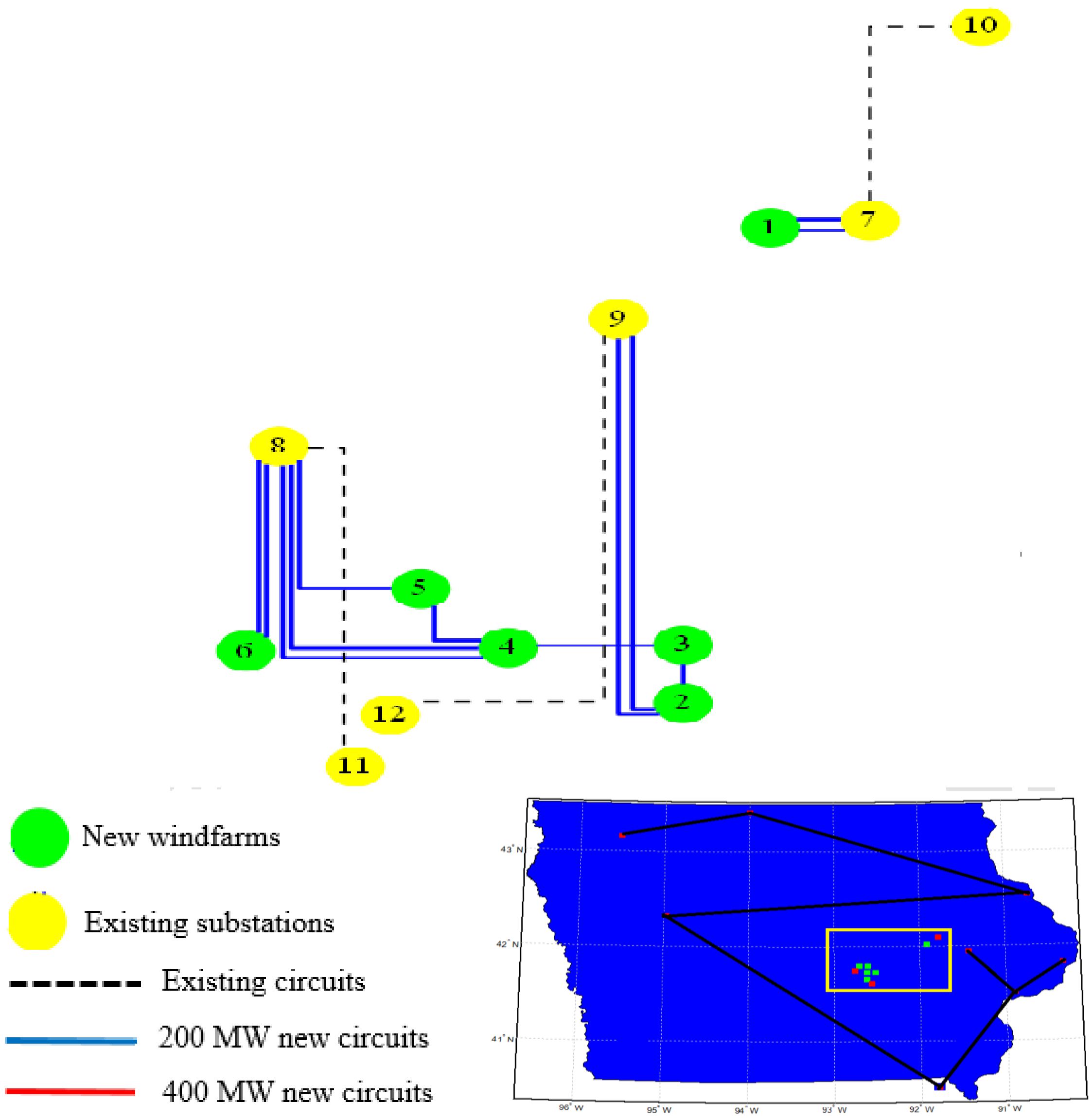

Figure 46: β= 0.2(Core-trajectory)

All designs are N-1 secure and all the wind farms are connected.

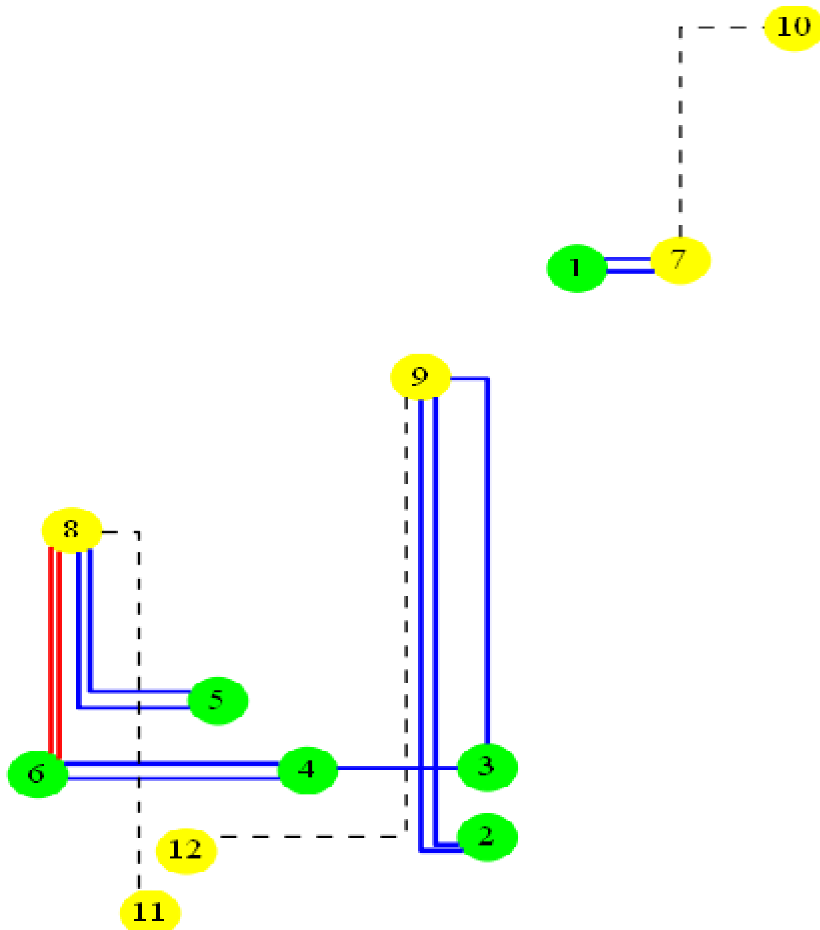

Figure 47: β= 0.9(Core-trajectory)

Validation

We validate these results using two different approaches. In both approaches, six deterministic designs (the optimal solutions for the six representative scenarios) are compared with an adaptive design obtained based values of β=0.4 and β=0.9.

The results of validation approach 1 are described first, while the results of validation approach 2 are described thereafter.

Approach #1

Figure 48 below provides the total costs across all scenarios for the adaptation-based design (β=0.4 and β=0.9) and different deterministic designs. Here, "total costs" refers to the sum of total investments (both the original investments (i.e. initial investment) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon.

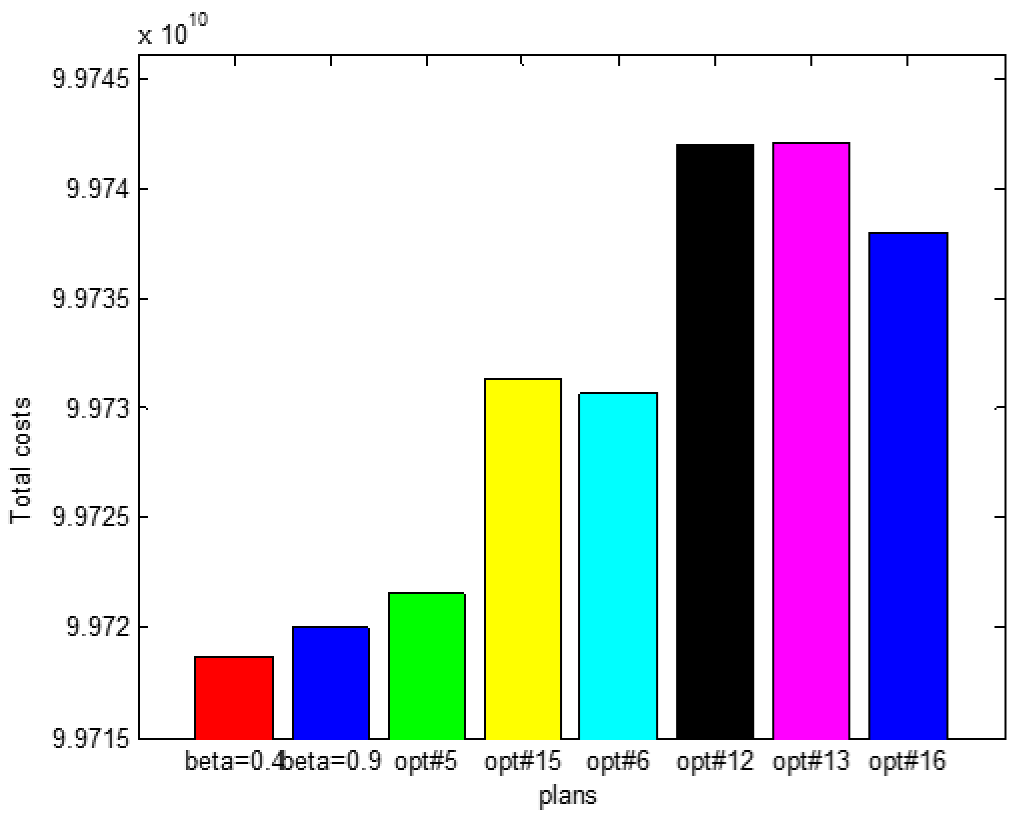

Figure 48: Average total costs across all scenarios for different deterministic and β designs

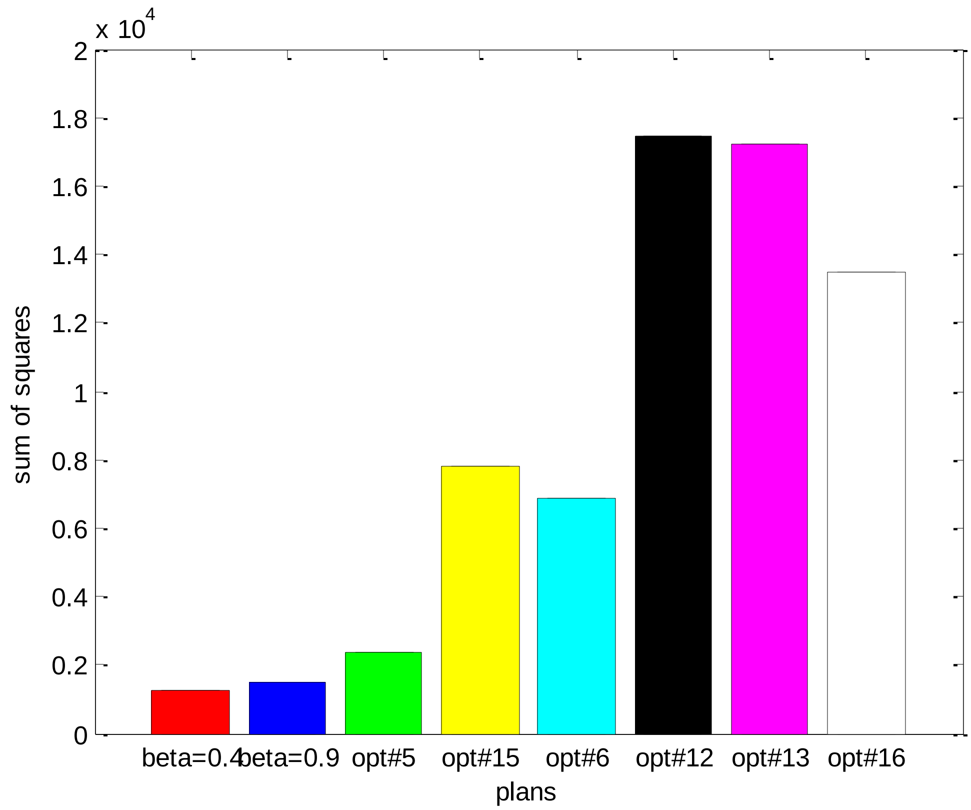

Figure 49: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 49 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

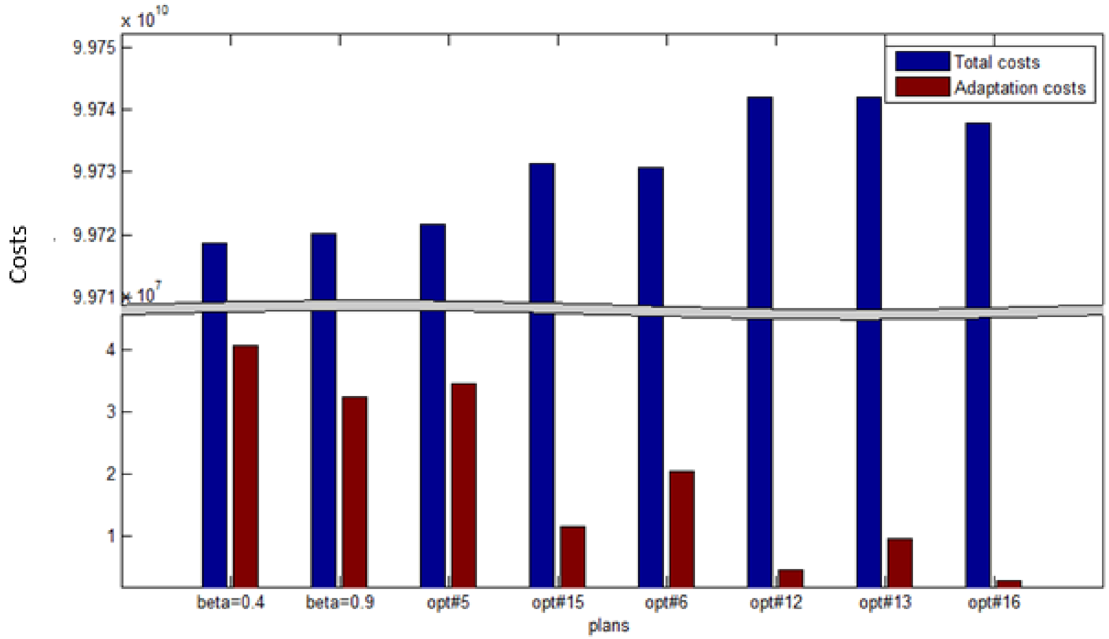

Figure 50: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 50 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig. 50 above that Opt#12 and Opt#16 have low adaptation costs but very high total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

Approach # 2

Figure 51 below provides the average total costs across all scenarios for the adaptation-based design (β=0.4 and β=0.9) and different deterministic designs. Here, "total costs" refers to the sum of total investments (both the original investments(i.e cost of coe-trajectory) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon.

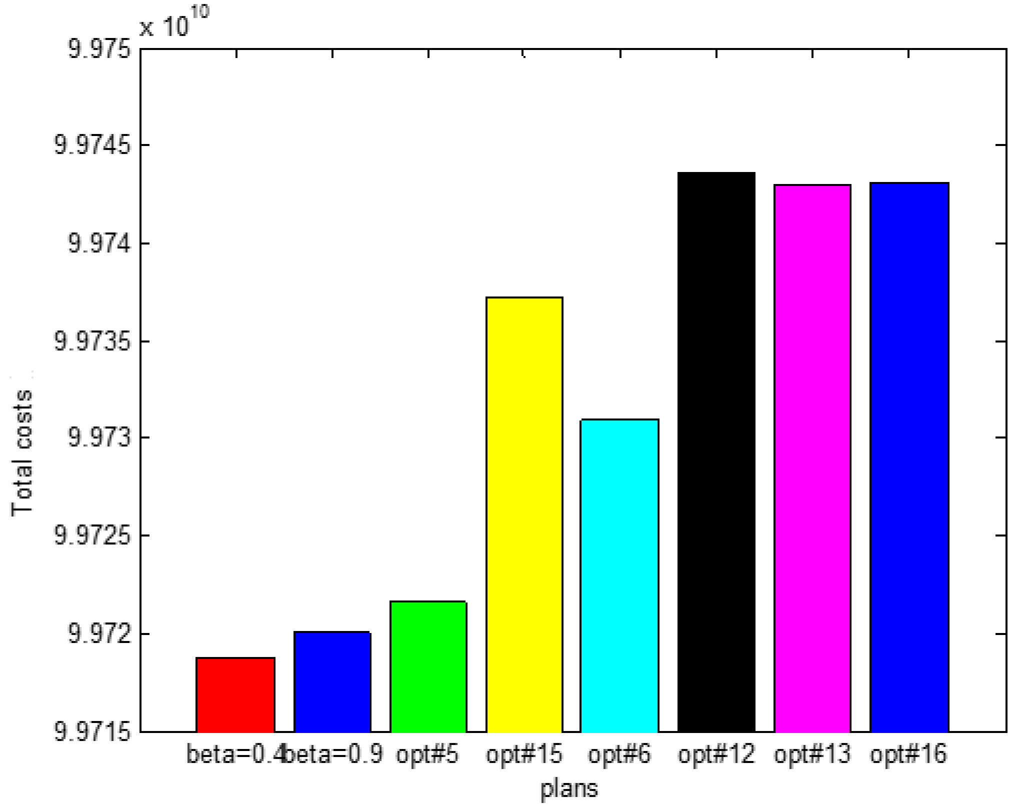

Figure 51: Average total costs across all scenarios for different deterministic and β designs

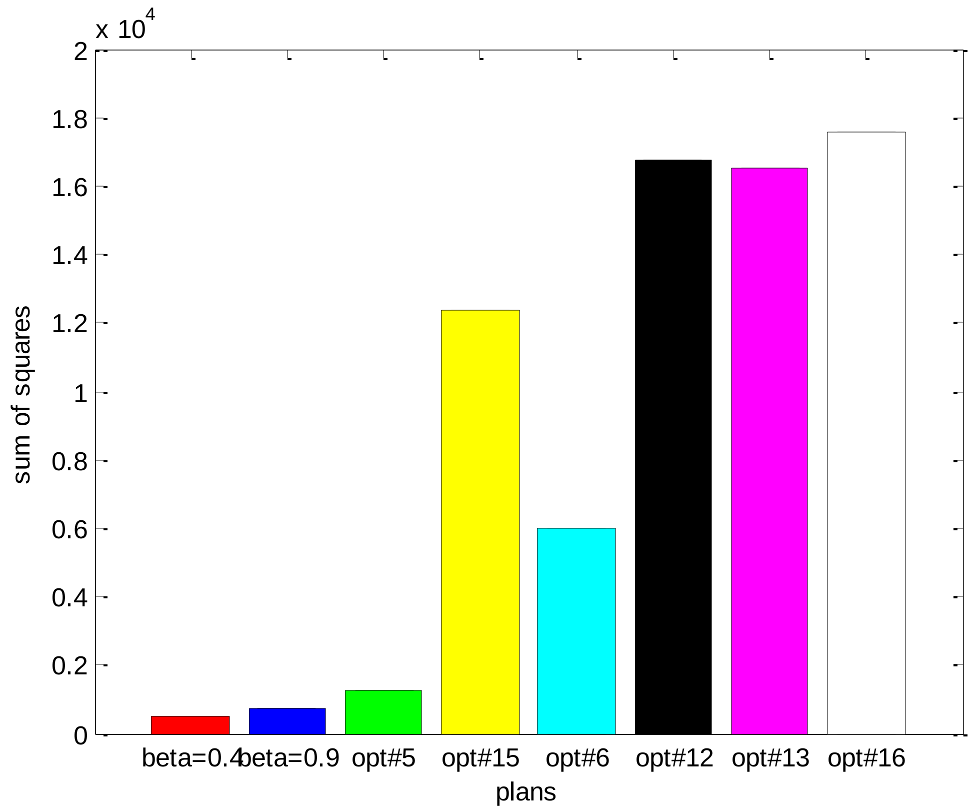

Figure 52: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 52 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

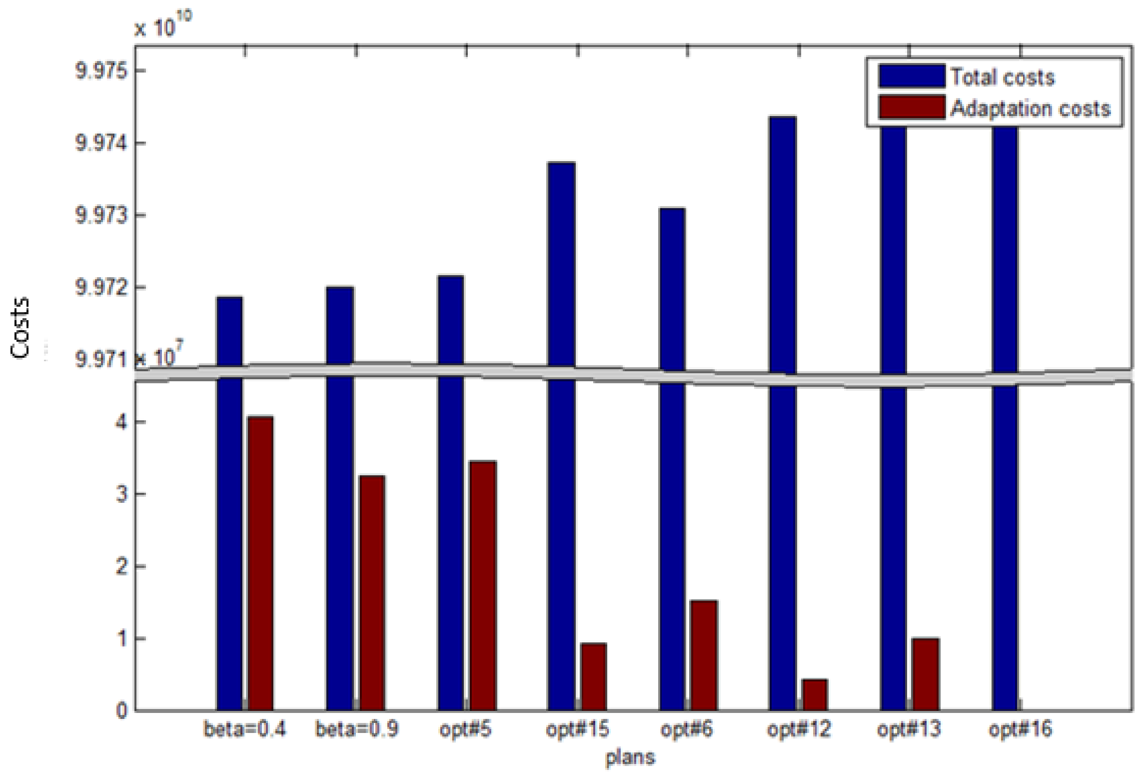

Figure 53: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 53 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig. 53 above that Opt# 16 has the lowest adaptation costs but has a very high total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

Case-study for wind group #3

The figure for wind cluster #3 can be located on the Iowa map in section 6.3

Table 23: Stage 1 wind-farm capacities

WF/MW	1	2	3	4	5	6
	191	199	199	198	200	199

Scenario generation

We divide the number of buses based on longitude and latitude data in to four areas {1,2} {3,4} {5} {6}.Therefore the 6 buses are divided four areas, we consider that each bus in an area can either increase by 150MW or not in the next stage. This gives a maximum of 16 (i.e 2^4) scenarios. In this case-study, 14 scenarios are used, we solve for the optimal investment for each scenario separately.

Scenario reduction

The scenario reduction technique is performed using hierarchical clustering technique using a dendrogram. The optimal solution is solved for all scenarios. The transmission similarity index is computed to measures the similarity between the transmission investments of two scenarios. The closer the value to one the stronger the similarities between scenarios.

Table 24: Optimal solutions for all scenarios

			Scenarios													
From	**To**	**MW**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	2	200	0	0	0	0	1	0	1	1	1	0	0	1	0	0
1	3	200	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	4	200	0	1	1	0	0	1	0	0	0	0	0	0	0	1
1	9	200	1	1	1	1	1	1	1	1	1	1	1	1	0	1
1	9	200	1	1	1	1	0	1	1	0	1	1	1	0	0	0
1	9	400	0	0	1	1	1	1	1	1	1	1	0	1	1	1
1	9	400	0	0	0	0	1	0	0	1	0	0	0	1	1	1
2	3	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	3	200	0	0	1	0	1	1	1	1	1	1	1	1	1	1
2	4	200	1	0	1	1	1	0	1	1	1	1	1	1	1	1
2	4	200	0	0	0	0	0	0	1	0	1	0	1	0	0	1
2	9	200	1	1	1	1	1	1	1	1	1	0	1	1	1	1
2	9	200	1	1	1	1	1	1	1	1	1	0	0	1	1	1
2	9	400	0	0	0	0	0	0	0	0	0	1	1	0	1	1

Table 24 continued

2	9	400	0	0	0	0	0	0	0	0	0	1	1	0	0	0
3	4	200	0	0	0	0	0	1	0	0	0	0	0	1	0	0
3	5	200	0	0	0	0	0	1	0	0	0	0	0	0	1	1
3	7	200	0	0	1	1	1	0	0	0	0	0	0	0	0	0
3	8	200	1	1	0	0	0	0	1	0	1	1	1	0	0	0
4	5	200	0	0	0	0	0	0	0	0	0	1	1	0	0	0
4	8	200	1	1	1	1	1	0	1	1	1	0	0	1	1	1
4	8	200	0	0	0	0	0	0	0	0	0	0	0	1	0	0
4	9	200	0	0	0	0	1	0	0	0	0	1	0	1	0	0
5	6	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	8	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	8	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	7	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	7	200	1	1	0	0	0	0	1	1	0	1	0	0	0	0
5	7	200	0	0	0	0	0	0	0	0	1	0	0	0	0	0
6	10	200	0	0	0	0	0	1	0	0	0	0	1	1	1	1
6	10	200	0	0	0	0	0	0	0	0	0	0	1	0	0	0

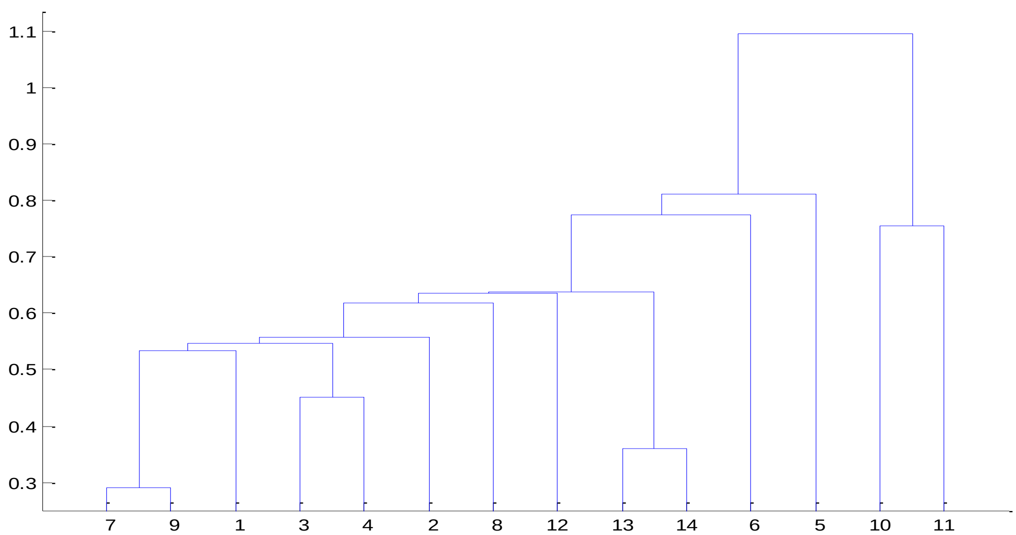

Figure 54: Dendrogram for scenario clustering for wind cluster #3

Cluster 1 {13, 14}

Cluster 2 {1, 2, 3, 4}

Cluster 3{7, 9}

Cluster 4 {8, 12}

Cluster 5 {10, 11}

Cluster 6 {6}

Cluster 7 {5}

A representative scenario is selected from each cluster and modeled explicitly in the adaptation formulation.

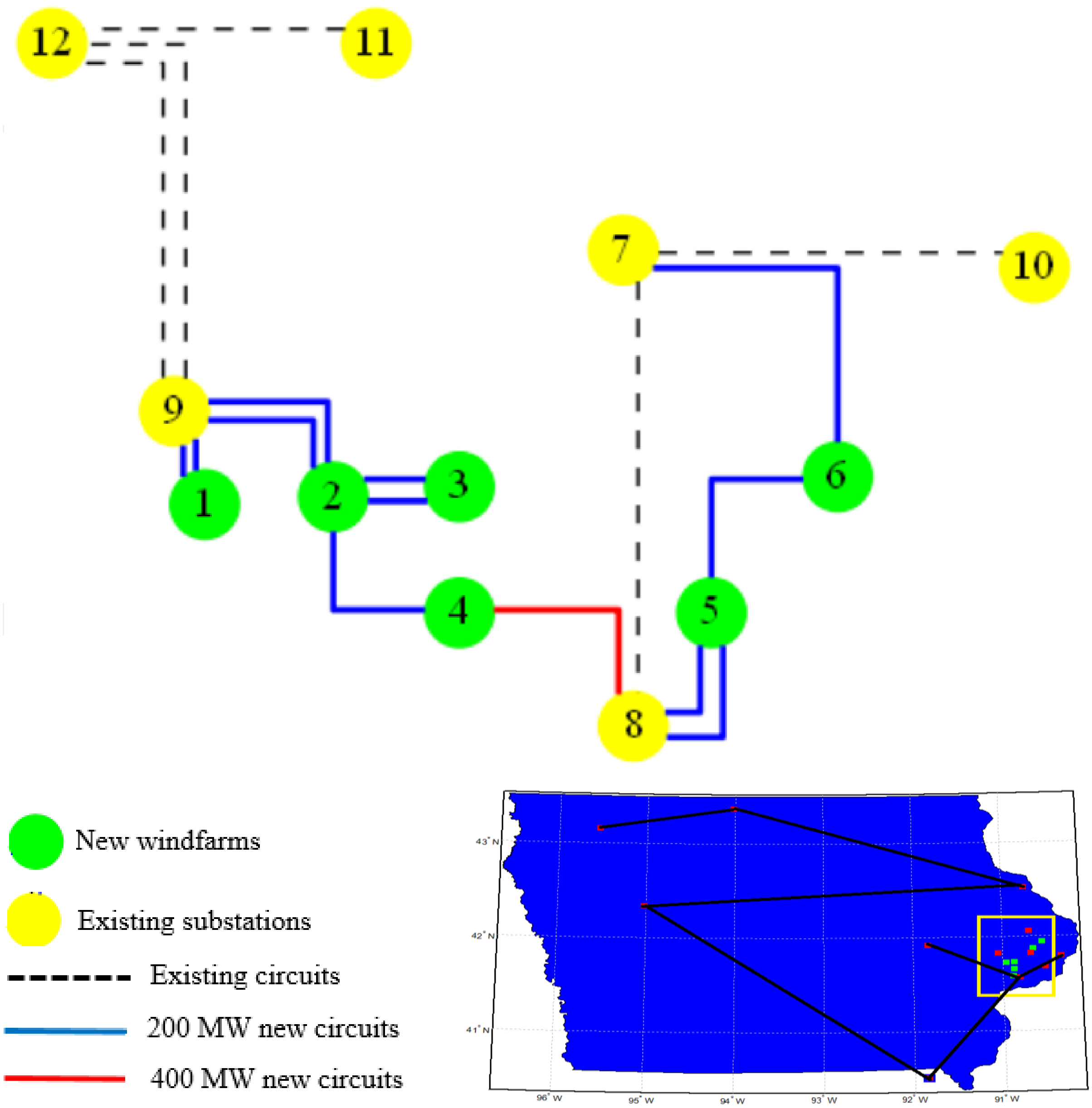

Figure 55: Design for β=0.7(Core-trajectory)

All designs are N-1 secure and all the wind farms are connected.

Validation

We validate these results using two different approaches. In both approaches, seven deterministic designs (the optimal solutions for the seven representative scenarios) are compared with an adaptive design obtained based values of $\beta=0.7$.

The results of validation approach 1 are described first, while the results of validation approach 2 are described thereafter.

Appoach # 1

Figure 56 below provides the average total costs across all scenarios for the adaptation-based design ($\beta=0.7$) and different deterministic designs. Here, "total costs" refers to the sum of total investments (both the original investments (i.e. initial investment) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. It can be seen in Fig.56 below that the adaptation based design has the lowest average total costs when compared to all other deterministic designs.

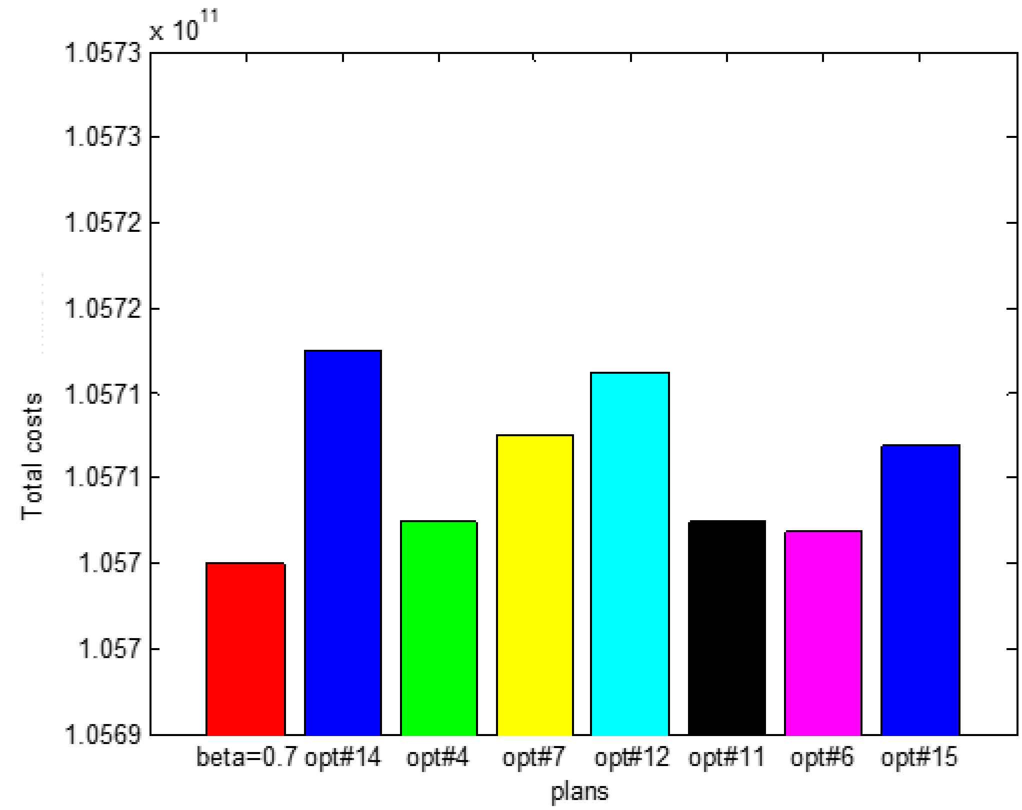

Figure 56: Average total costs across all scenarios for different deterministic and β designs

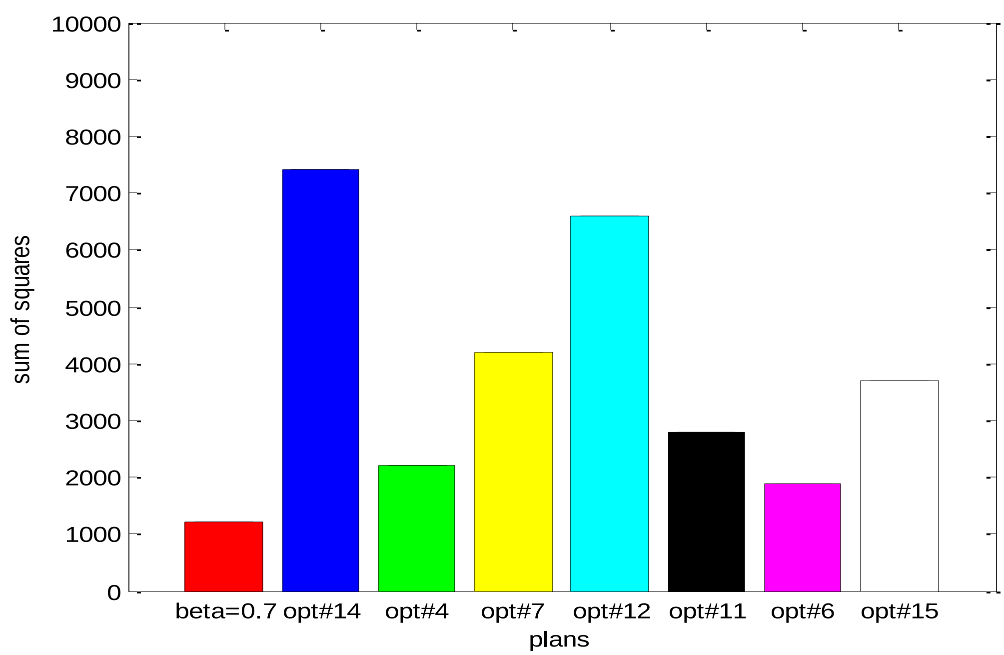

Figure 57: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 57 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

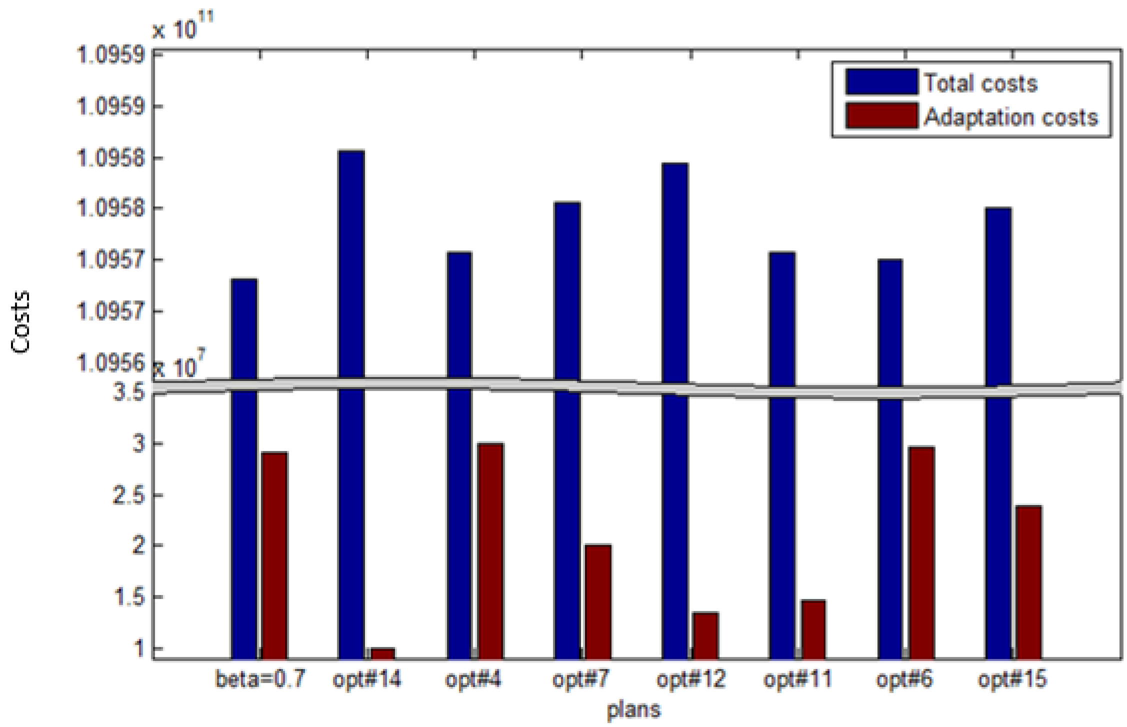

Figure 58: Average adaptation costs and total costs for different deterministic and β designs

The vertical line in Fig. 58 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig. 58 that Opt#14 has the lowest adaptation cost but has the highest total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

Approach # 2

Figure 59 below provides the average total costs across all scenarios for the adaptation-based design (β=0.7) and different deterministic designs .Here, "total costs" refers to the sum of total investments (both the original investments (i.e. cost of core-trajectory) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. It can be seen in Fig. 59 below that the adaptation based design has the lowest average total costs when compared to all other deterministic designs.

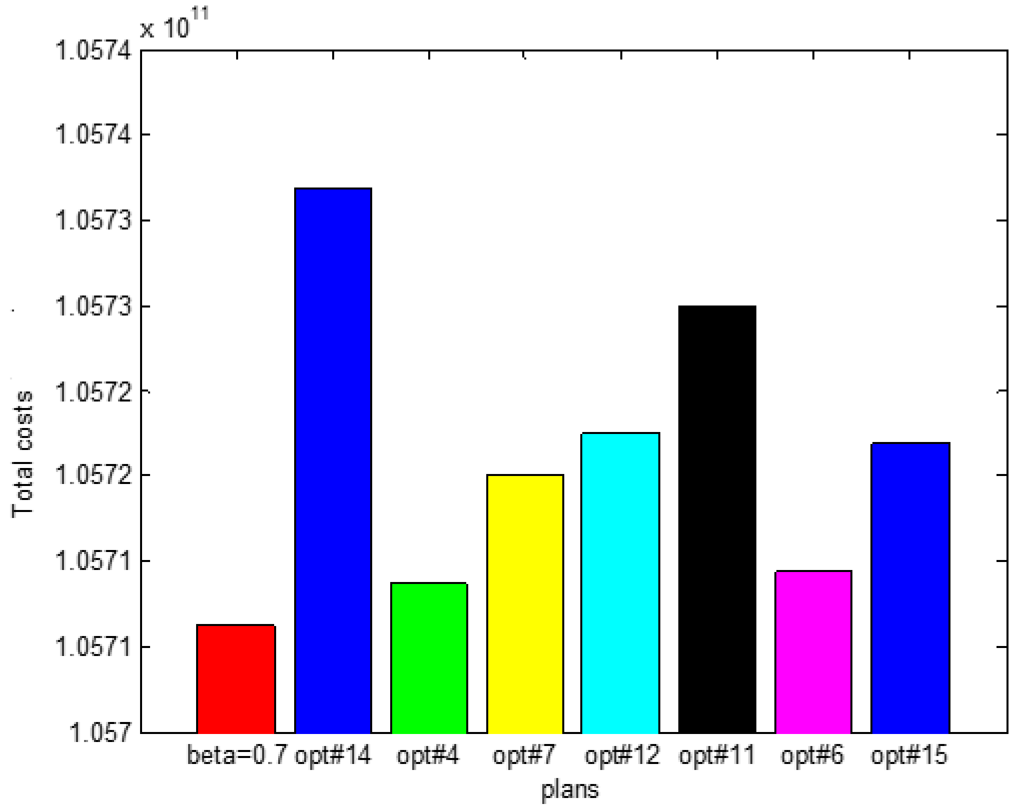

Figure 59: Average total costs across all scenarios for different deterministic and β designs

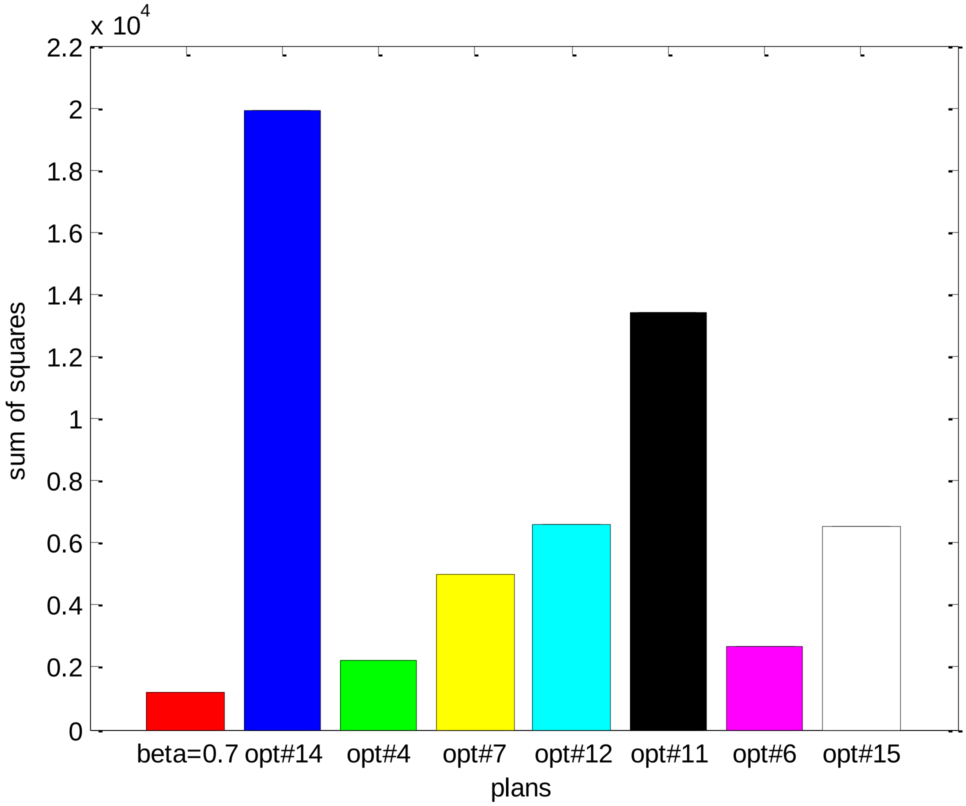

Figure 60: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 60 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

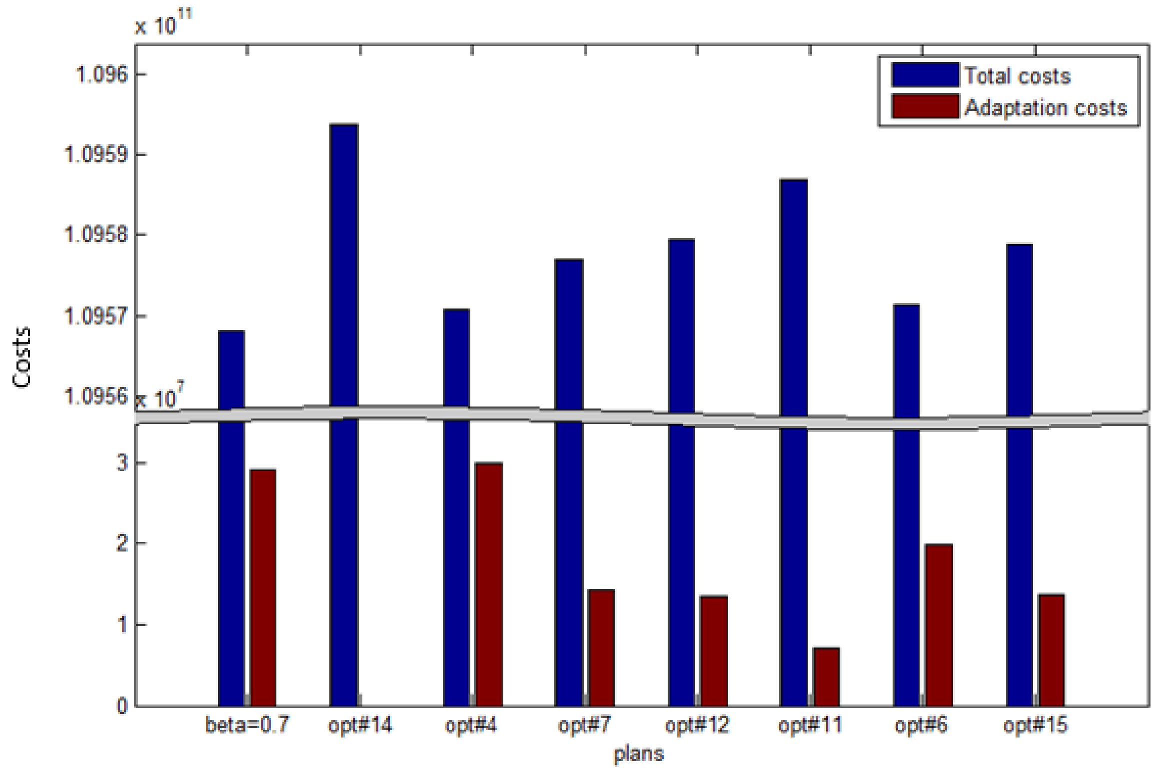

Figure 61: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 61 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig. 61 above that Opt#14 has the lowest adaptation cost but has the highest total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

Case-study for wind group #4

The figure for wind cluster #4 can be located on the Iowa map in section 6.3.

Table 25: Stage 1 wind-farm capacities

WF/MW	1	2	3	4	5	6	7	8	9
	200	199	197	196	194	199	199	200	195

Scenario generation

We divide the number of buses based on longitude and latitude data in to four areas {{1,2} {3,4,7} {5,9} {6,8}.Therefore the 9 buses are divided four areas, we consider that each bus in an area can either increase by 150MW or not in the next stage. This gives a maximum of (i.e 2^4) 16 scenarios. In this case-study, 15 scenarios are used, we solve for the optimal investment for each scenario separately.

Scenario reduction

The scenario reduction technique is performed using hierarchical clustering technique using a dendrogram. The optimal solution is solved for all scenarios. The transmission similarity index is computed to measures the similarity between the transmission investments of two scenarios. The closer the value to one the stronger the similarities between scenarios.

Table 26: Optimal solutions for all scenarios

			Scenarios															
From	**To**	**MW**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	2	200	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	1
1	2	200	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
1	3	200	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
1	7	200	1	0	1	1	1	0	0	1	0	1	1	0	1	0	1	1
1	7	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	10	200	1	1	1	0	0	1	0	0	0	0	0	1	0	1	0	0
1	10	200	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0
1	11	200	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0
1	11	200	0	0	0	1	1	1	1	1	1	1	1	0	1	0	1	0
2	8	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
2	10	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	10	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	11	200	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1
3	4	200	1	0	0	1	0	0	0	0	0	0	0	1	0	1	0	1
3	4	200	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1
3	5	200	0	0	0	0	1	0	1	0	0	1	1	1	1	1	1	0
3	7	200	0	0	1	1	0	1	0	1	1	0	0	0	0	0	0	1
3	7	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
3	10	200	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1

Table 26 continued

3	10	200	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1
3	10	400	0	0	0	1	0	1	0	0	0	0	1	1	1	1	1	1
3	10	400	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1
4	5	200	0	1	1	0	1	1	1	1	1	1	1	0	1	0	1	1
4	7	200	0	1	0	0	0	0	1	0	0	0	0	1	0	1	0	0
4	9	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
4	10	200	0	0	1	1	0	0	1	0	0	0	0	1	0	1	0	0
4	12	200	0	0	0	1	1	1	0	1	1	1	1	0	1	0	1	0
4	12	200	0	0	0	0	1	1	0	1	1	1	1	0	1	0	1	0
5	9	200	1	1	1	1	0	1	1	1	1	0	1	1	1	1	1	1
5	9	200	1	1	1	1	0	1	1	1	1	0	1	0	0	0	0	1
5	10	200	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1
6	8	200	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0
6	9	200	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
6	9	200	1	1	1	1	0	0	1	1	0	0	0	1	0	1	1	0
6	10	200	0	1	0	0	1	1	1	1	1	1	1	0	1	0	1	1
6	10	200	0	0	0	0	1	0	0	0	1	1	0	0	1	0	0	1
7	12	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	12	200	1	1	0	0	1	1	1	0	1	1	1	1	1	1	1	1
8	9	200	0	1	0	0	0	0	1	1	0	1	0	0	1	1	0	1
8	9	200	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	10	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
8	10	200	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1
8	10	400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
8	10	400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
9	10	200	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
9	10	200	0	1	1	0	1	1	1	1	0	0	0	0	0	0	0	1
9	10	400	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1

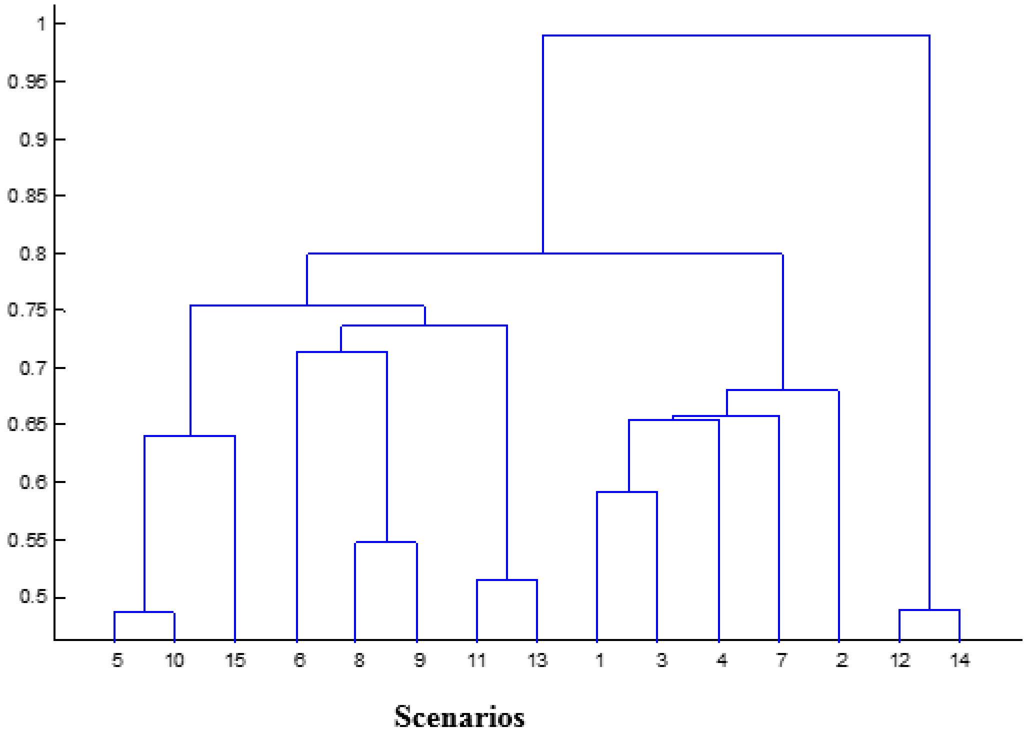

Figure 62: Dendrogram for scenario clustering for wind cluster #4

The scenarios were divided into 4 clusters

Cluster 1 {6,8,9,11,13}

Cluster 2{5,10,15}

Cluster 3 {1,2,3,4,7}

Cluster 4 {12,14}

A representative scenario is selected from each cluster and modeled explicitly in the adaptation formulation.

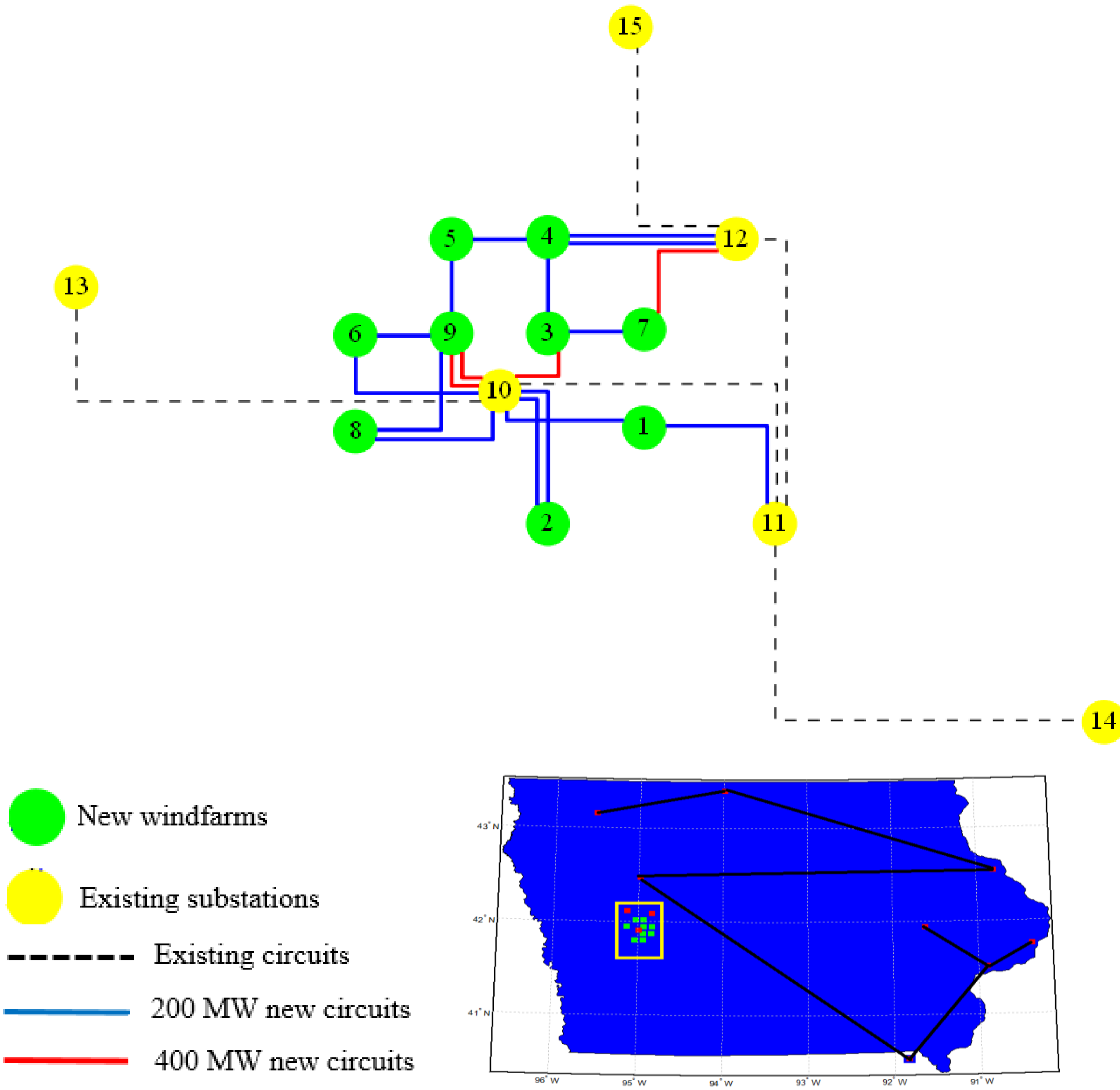

Figure 63: Design for β= 0.75(Core-trajectory)

All designs are N-1 secure and all the wind farms are connected.

Validation

We validate these results using two different approaches. In both approaches, four deterministic designs (the optimal solutions for the four representative scenarios) are compared with an adaptive design obtained based values of β=0.75.

The results of validation approach 1 is presented first and the results of validation approach 2 are presented next.

Approach # 1

Figure 64 below provides the average total costs across all scenarios for the adaptation-based design (β=0.75) and different deterministic designs. Here, "total costs" refers to the sum of total investments (both the original investments (i.e. initial investments) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. It can be seen in Fig. 64 below that the adaptation based design has the lowest average total costs when compared to all other deterministic designs.

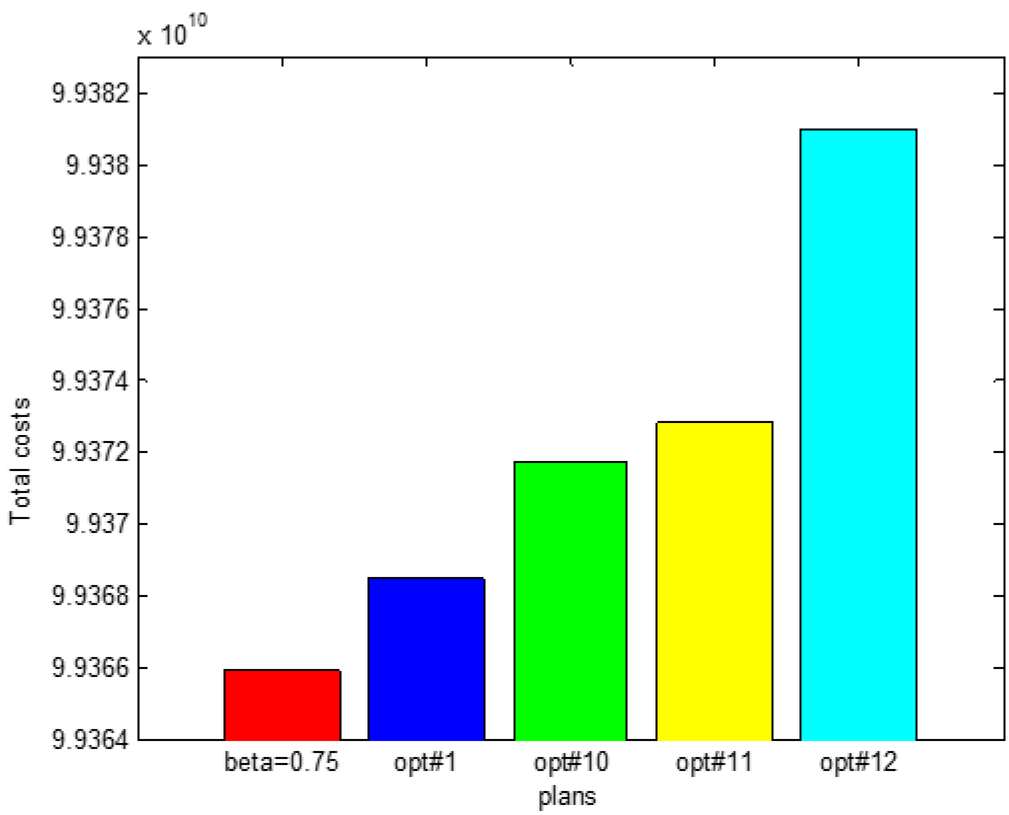

Figure 64: Average total costs across all scenarios for different deterministic and beta designs

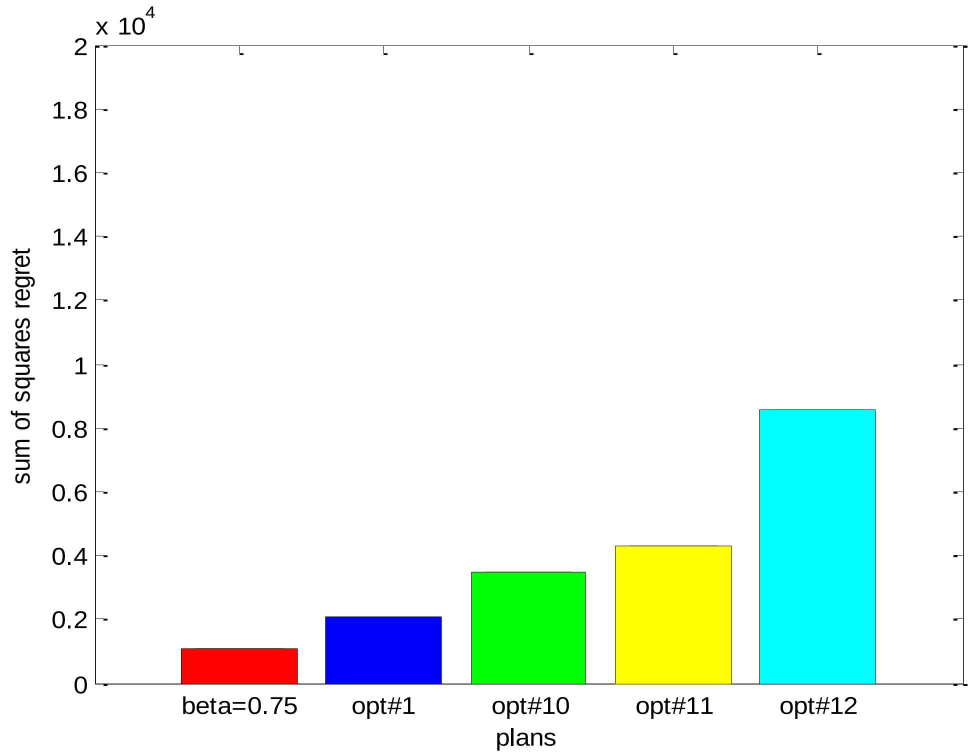

Figure 65: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 65 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

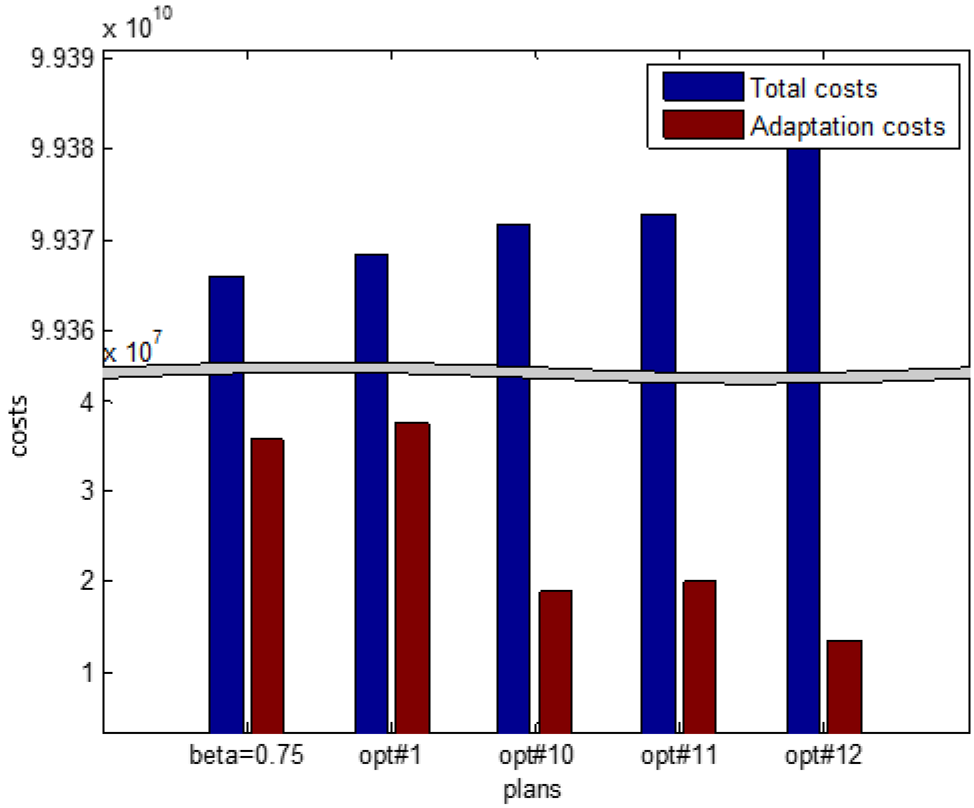

Figure 66: Average adaptation costs and total costs for different deterministic and β design

The horizontal grey line in Fig. 66 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig. 66 above that Opt#12 has the lowest adaptation costs but has the highest total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

Approach #2

Figure 67 below provides the total costs across all scenarios for the adaptation-based design (β=0.75) and different deterministic designs. Here, “total costs” refers to the sum of total investments (both the original investments (i.e. cost of core-trajectory) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. It can be seen in Fig.67 below that the adaptation based design has the lowest average total costs when compared to all other deterministic designs.

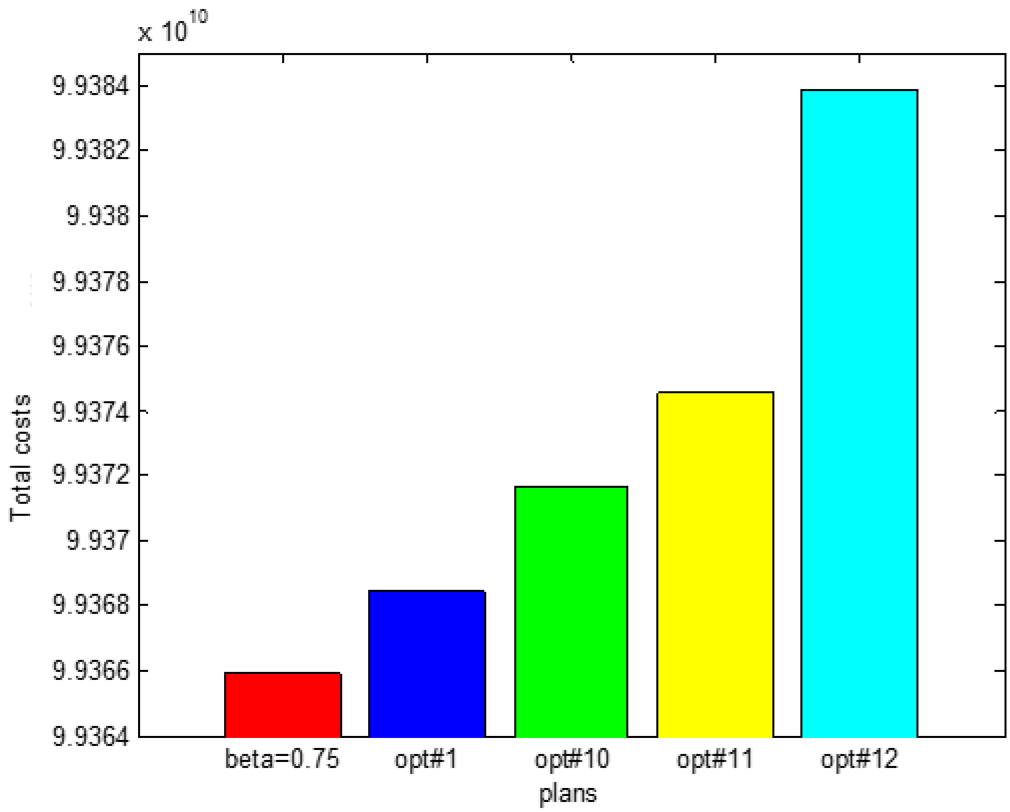

Figure 67: Average total costs across all scenarios for different deterministic and β designs

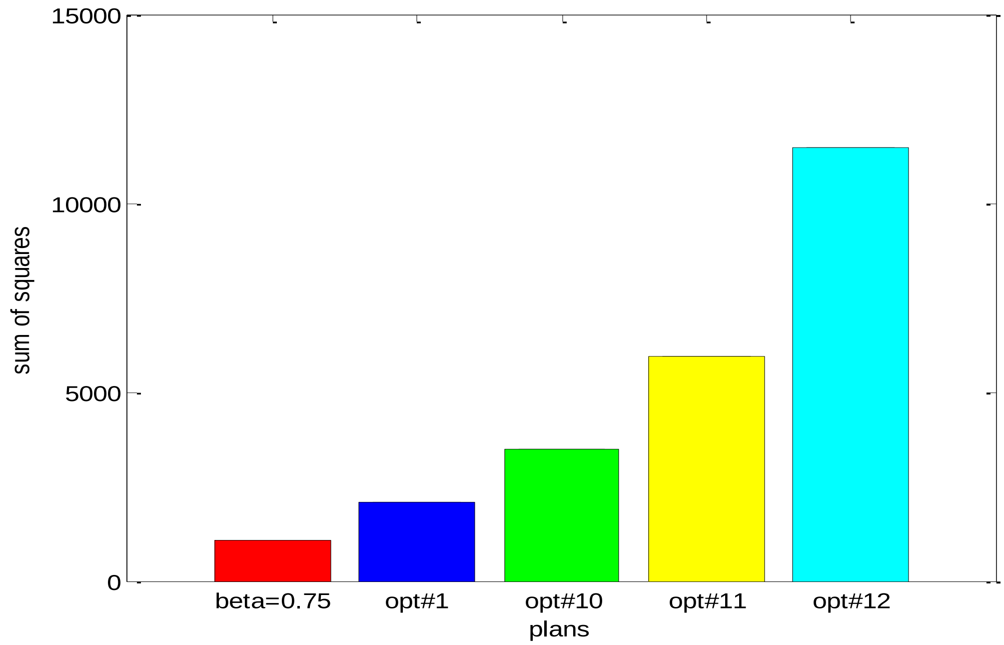

Figure 68: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 68 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

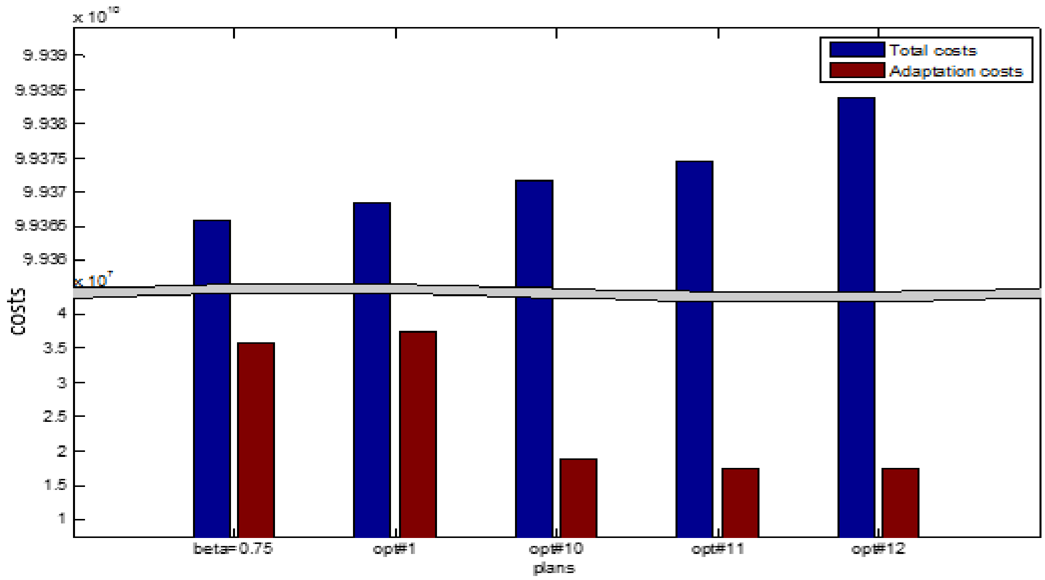

Figure 69: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 69 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig. 69 above that Opt#12 has the lowest adaptation costs but has the highest total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

Case-study for wind cluster #5

The figure for wind cluster #5 can be located on the Iowa map in section 6.3.

Table 27: Stage 1 wind-farm capacities

WF/MW	1	2	3	4	5	6	7	8	9	10	11
	191	191	195	198	185	191	196	196	197	190	200

Scenario generation

We divide the number of buses based on longitude and latitude data in to four areas {5} {9,10} {3,7,8,11} {1,2,4,6}.Therefore the 11 buses are divided into four areas, we consider that each bus in an area can either increase by 150MW or not in the next stage. This gives a maximum of (i.e 2^4) 16 scenarios. In this case-study, 16 scenarios are used, we solve for the optimal investment for each scenario separately.

Scenario reduction

The scenario reduction technique is performed using hierarchical clustering technique using a dendrogram. The optimal solution is solved for all scenarios. The transmission similarity index is computed to measures the similarity between the transmission investments of two scenarios. The closer the value to one the stronger the similarities between scenarios.

Table 28: Optimal solutions for all scenarios

			Scenarios															
From	**To**	**MW**	**1***	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	3	200	1	1	1	1	1	1	0	0	1	1	1	0	0	1	0	0
1	4	200	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1
1	6	200	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	1
1	6	200	1	0	1	1	1	1	0	1	0	0	1	1	1	0	1	1
1	12	200	1	0	0	1	1	1	0	0	0	0	0	1	1	0	1	1
1	12	200	0	1	0	0	0	0	1	0	1	1	0	0	1	1	1	1
1	12	400	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
2	4	200	0	1	0	0	0	0	1	0	1	1	0	0	1	1	1	1
2	6	200	1	1	1	1	1	1	0	1	1	1	1	1	0	0	1	0
2	12	200	1	1	1	1	1	1	0	1	1	1	1	1	0	0	1	0
2	12	200	0	1	0	0	0	0	1	0	1	1	0	0	1	1	1	1
2	12	400	0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	1
2	16	200	0	0	1	0	0	0	1	0	0	0	0	1	1	0	0	1
3	8	200	0	1	1	0	0	0	1	1	1	1	1	1	1	1	1	0
3	11	200	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0
3	11	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	12	200	0	1	1	0	0	0	0	1	1	1	1	1	0	1	1	1
3	12	200	0	0	0	0	0	0	1	1	0	0	1	0	1	0	1	0
3	12	400	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	6	200	0	1	1	0	0	0	0	1	1	1	1	0	0	1	0	0

Table 28 continued

4	6	200	1	0	1	1	1	1	1	1	0	0	1	1	0	0	1	1
4	7	200	1	0	0	1	1	0	1	1	0	0	1	1	0	0	1	1
4	7	200	1	1	1	1	1	1	0	1	1	1	1	0	1	1	0	0
4	8	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	11	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	12	200	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
4	12	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	12	400	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	16	200	1	1	1	1	0	0	1	0	1	0	1	0	1	0	0	0
5	13	200	1	1	1	1	0	0	1	0	1	0	1	0	1	0	0	0
5	13	200	0	0	0	0	1	1	0	1	0	1	0	1	0	1	1	1
5	13	400	0	0	0	0	1	1	0	1	0	1	0	1	0	1	1	1
5	13	400	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
7	8	200	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
7	11	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	12	200	0	1	1	0	0	0	1	0	1	1	0	1	1	1	1	1
7	16	400	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1
8	12	200	0	0	1	0	0	0	1	1	0	0	1	1	1	0	1	1
8	16	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	10	200	0	0	0	1	0	1	0	0	1	0	1	1	1	0	0	1
9	10	200	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
9	14	200	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
9	14	200	0	0	0	1	0	1	0	0	1	0	1	1	1	1	0	1
9	14	400	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
9	14	400	1	1	1	0	1	0	1	1	0	1	0	0	0	1	1	0
10	14	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	14	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	15	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	15	200	0	0	1	0	0	1	1	1	0	0	1	1	1	0	0	1
11	12	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
11	12	400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

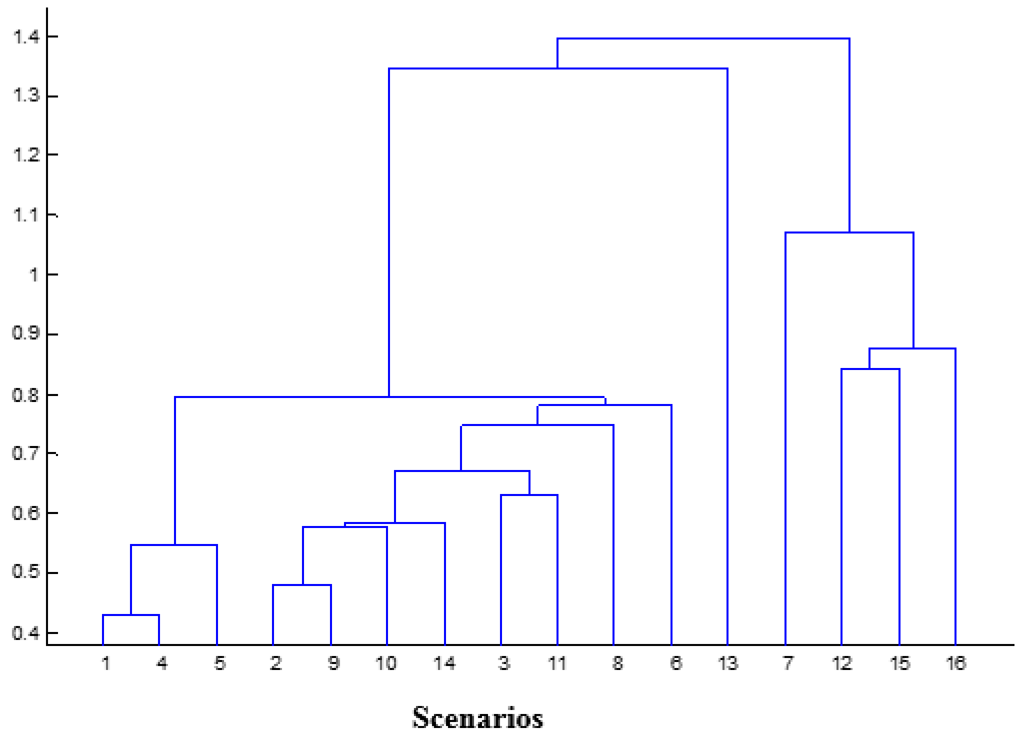

Figure 70: Dendrogram for scenario clustering for wind cluster #5

Cluster 1 { 1,4,5}

Cluster 2 {2, 9, 10, 14}

Cluster 3 { 3,11,8,6}

Cluster 4 {7}

Cluster 5 {13}

Cluster 6 {12,15,16}

A representative scenario is selected from each cluster and modeled explicitly in the adaptation formulation.

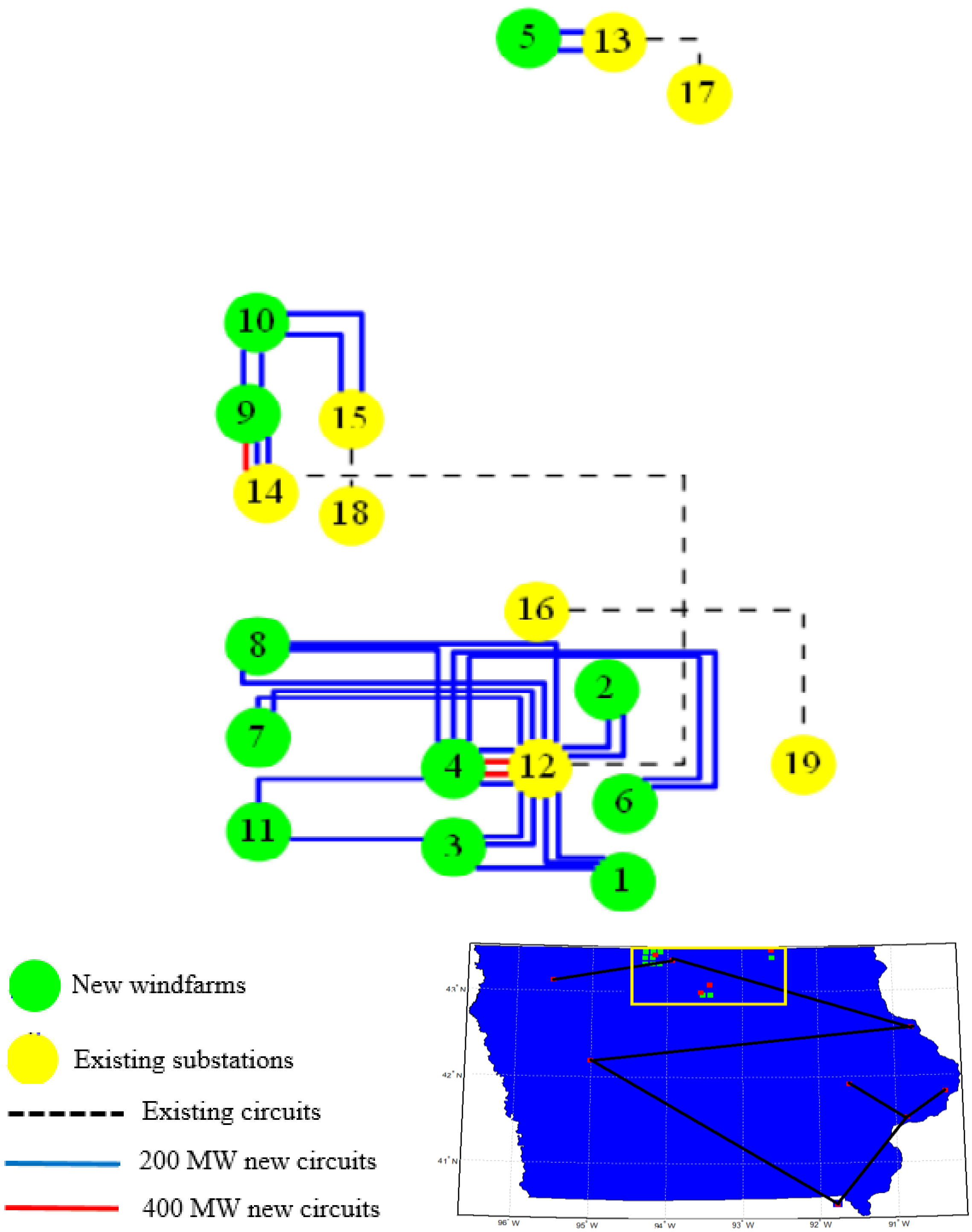

Figure 71: Design for β= 1(Core-trajectory)

Validation

We validate these results using two different approaches. In both approaches, six deterministic designs (the optimal solutions for the six representative scenarios) are compared with an adaptive design obtained based values of β=1.

The results of validation approach 1 is presented first and the results of validation approach 2 is presented next.

Approach # 1

Figure 72 below provides the total costs across all scenarios for the adaptation-based design (β=0.8) and different deterministic designs. Here, "total costs" refers to the sum of total investments (both the original investments (i.e. initial investment) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. It can be seen in Fig.72 below that the adaptation based design has the lowest average total costs when compared to all other deterministic designs.

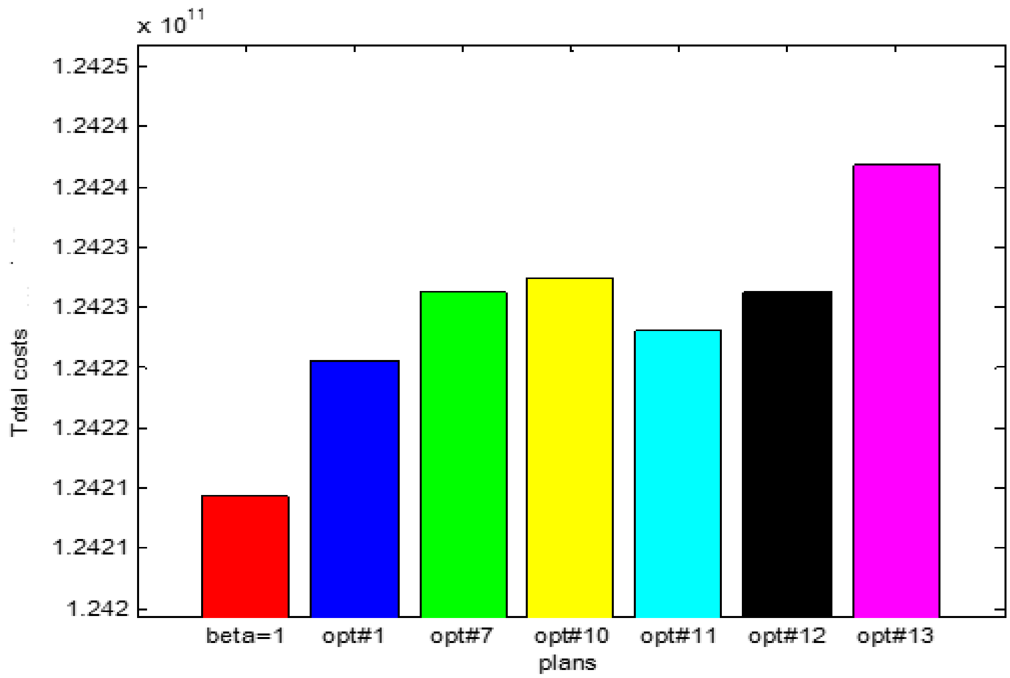

Figure 72: Average total costs across all scenarios for different deterministic and β designs

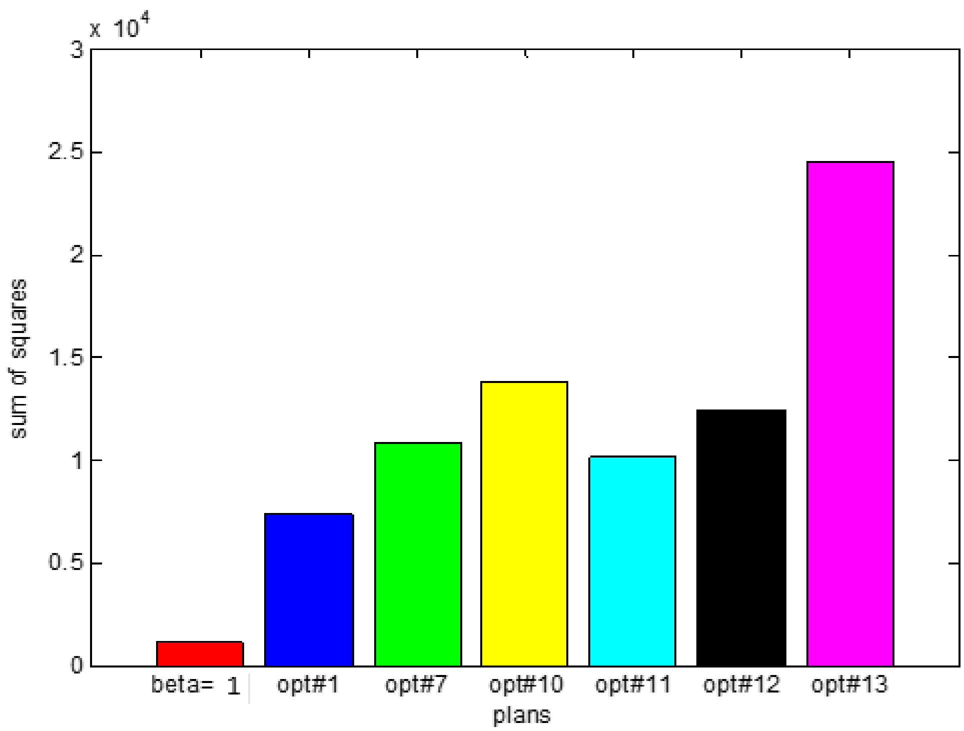

Figure 73: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig.73 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the figure above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

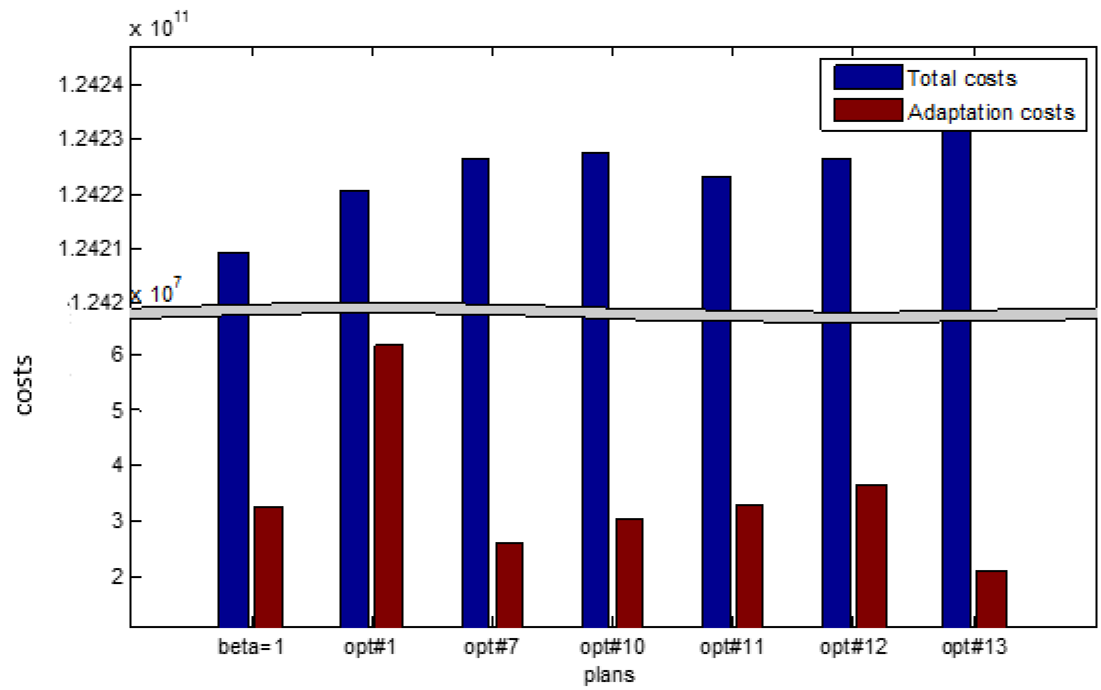

Figure 74: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 74 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. Opt#13 has the lowest adaptation costs but has the highest total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

Approach # 2

Figure 75 below provides average total costs across all scenarios for the adaptation-based design (β=1) and different deterministic designs. Here, "total costs" refers to the sum of total investments (both the original investments (i.e. cost of core-trajectory) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. It can be seen in Fig. 75 below that the adaptation based design has the lowest average total costs when compared to all other deterministic designs.

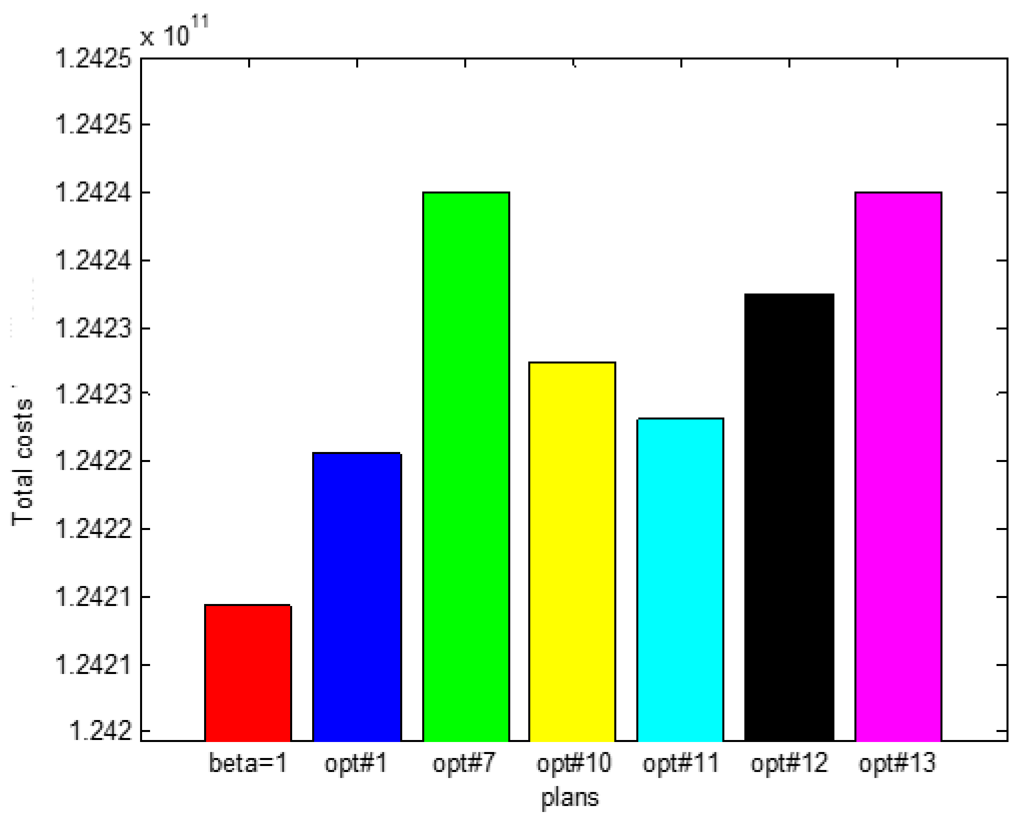

Figure 75: Average total costs across all scenarios for different deterministic and β designs

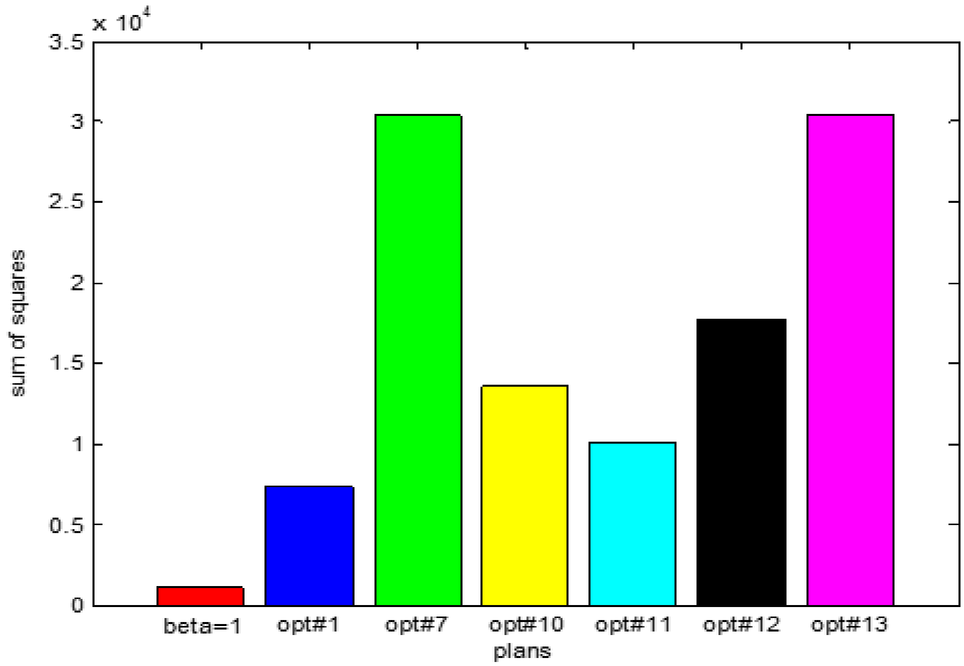

Figure 76: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 76 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the Fig. 77 above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

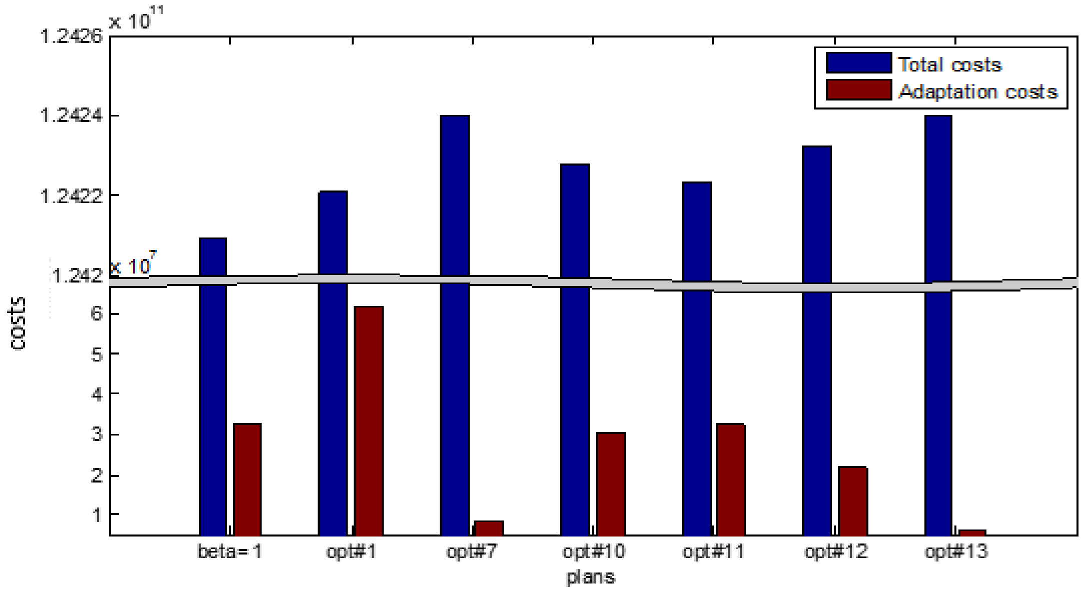

Figure 77: Average adaptation costs and total costs for different deterministic and β designs

The horizontal grey line in Fig. 77 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig. 77 above that Opt#13 has the lowest adaptation costs but has the total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

Case-study for wind group #6

The figure for wind cluster #6 can be located on the Iowa map in section 6.3.

Table 29: Stage 1 wind-farm capacities

WF/MW	1	2	3	4	5	6	7	8	9	10	11
	200	193	199	188	200	192	200	200	200	199	192

Scenario generation

We divide the number of buses based on longitude and latitude data in to four areas {5,6,7} {2,3,4} {10,11} {1,8,9}.Therefore the 11 buses are divided into four areas, we consider that each bus in an area can either increase by 150MW or not in the next stage. This gives a maximum of 16 (i.e. 2^4) scenarios. In this case-study, 16 scenarios are used.

Scenario reduction

The scenario reduction technique is performed using hierarchical clustering technique using a dendrogram. The optimal solution is solved for all scenarios. The transmission similarity index is computed to measures the similarity between the transmission investments of two scenarios. The closer the value to one the stronger the similarities between scenarios.

Table 30: Optimal solutions for all scenarios

			Scenarios															
From	**To**	**MW**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	15	200	0	0	0	1	1	1	1	1	0	1	0	1	0	1	1	1
1	17	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	17	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	3	200	0	0	1	0	0	0	0	1	0	0	1	1	1	0	1	1
2	4	200	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
2	16	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	16	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	16	400	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0
3	16	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	16	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	5	200	1	0	1	1	1	1	1	1	0	0	1	1	0	0	0	0
4	14	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	14	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	14	200	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
5	14	200	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
5	16	200	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1
5	16	200	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
6	7	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
6	7	200	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
6	14	200	1	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0

Table 30 continued

6	14	200	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
6	16	200	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1
6	16	200	0	1	0	1	1	0	0	0	1	1	1	1	1	1	1	1
7	13	200	0	1	0	0	0	0	0	0	1	1	0	0	1	1	1	1
7	16	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	16	200	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1
8	9	200	0	0	0	1	1	1	1	1	0	1	0	1	0	1	1	1
8	15	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
8	15	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	15	400	0	0	0	1	1	0	1	1	0	1	0	1	0	1	1	1
9	15	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	15	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	12	200	1	1	1	1	1	0	1	1	0	1	0	0	0	0	1	0
10	12	200	0	0	0	0	0	1	1	0	1	0	1	1	1	1	0	1
10	13	200	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	13	200	0	0	0	0	0	1	0	0	1	0	1	1	1	1	0	1
11	12	200	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1
11	12	200	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1
11	12	400	0	0	0	0	0	1	1	0	1	0	0	1	1	1	0	1
11	12	400	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0
2	14	200	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
4	14	400	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1

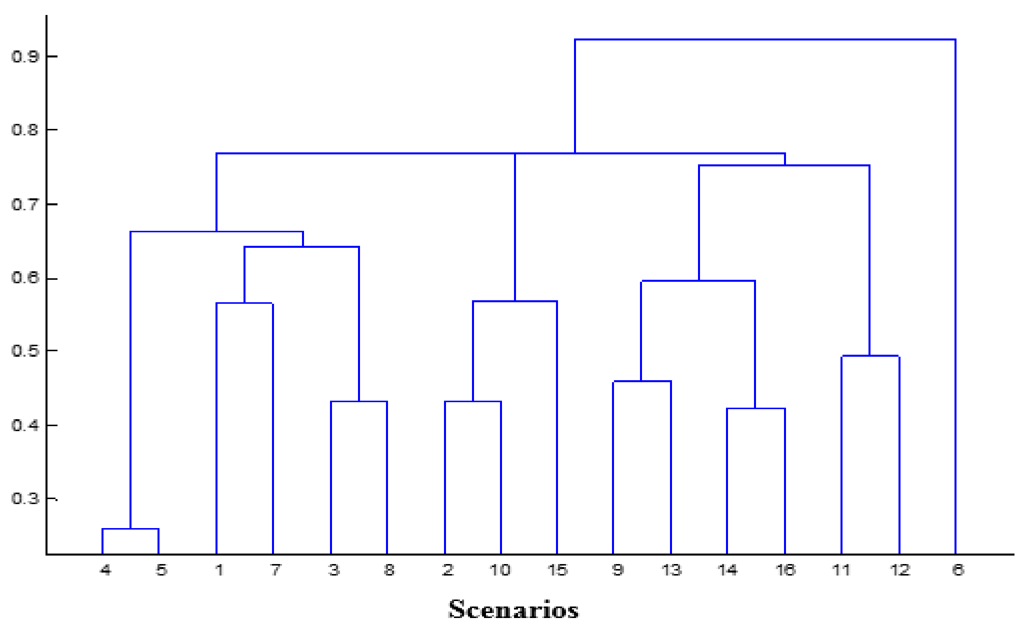

Figure 78: Dendrogram for scenario clustering for wind cluster #6

Cluster 1 {4,5}

Cluster 2 {1, 7, 3, 8}

Cluster 3 {2, 10, 15}

Cluster 4 {9, 13, 14, 16}

Cluster 5 {11, 12}

Cluster 6 {6}

A representative scenario is selected from each cluster and modeled explicitly in the adaptation formulation.

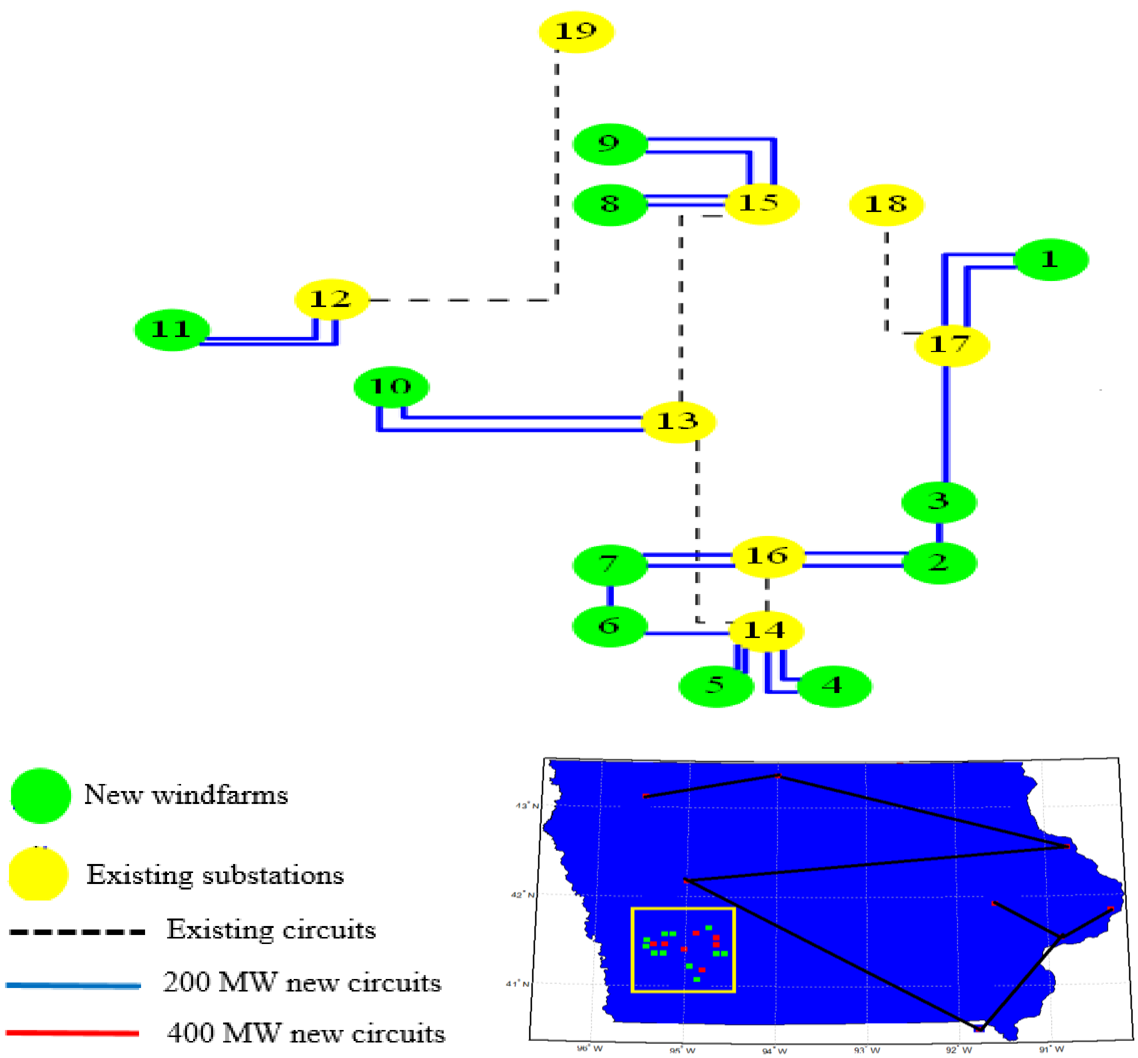

Figure 79: Design for β= 0.8(Core-trajectory)

All designs are N-1 secure and all the wind farms are connected.

Validation

We validate these results using two different approaches. In both approaches, six deterministic designs (the optimal solutions for the six representative scenarios) are compared with an adaptive design obtained based values of β=0.8.

The results of validation approach 1 is described first and the results for validation approach 2 is described next.

Approach # 1

Figure 80 below provides the average total costs across all scenarios for the adaptation-based design (β=0.8) and different deterministic designs. Here, “total costs” refers to the sum of total investments (both the original investments (i.e. initial investments) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. The adaptation based design has the lowest total costs when compared to all other deterministic designs. It can be seen in Fig. 80 below, that the adaptation based design has the lowest average total costs when compared to all other deterministic designs.

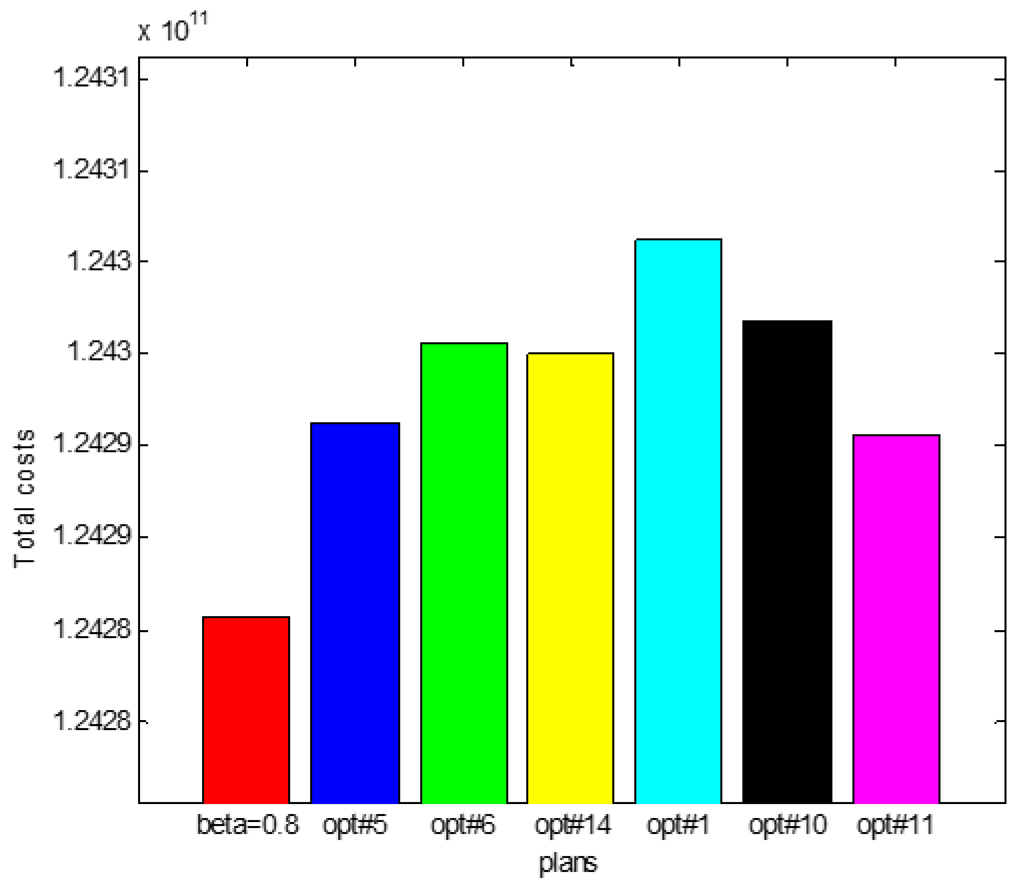

Figure 80: Average total costs across all scenarios for different deterministic and β designs

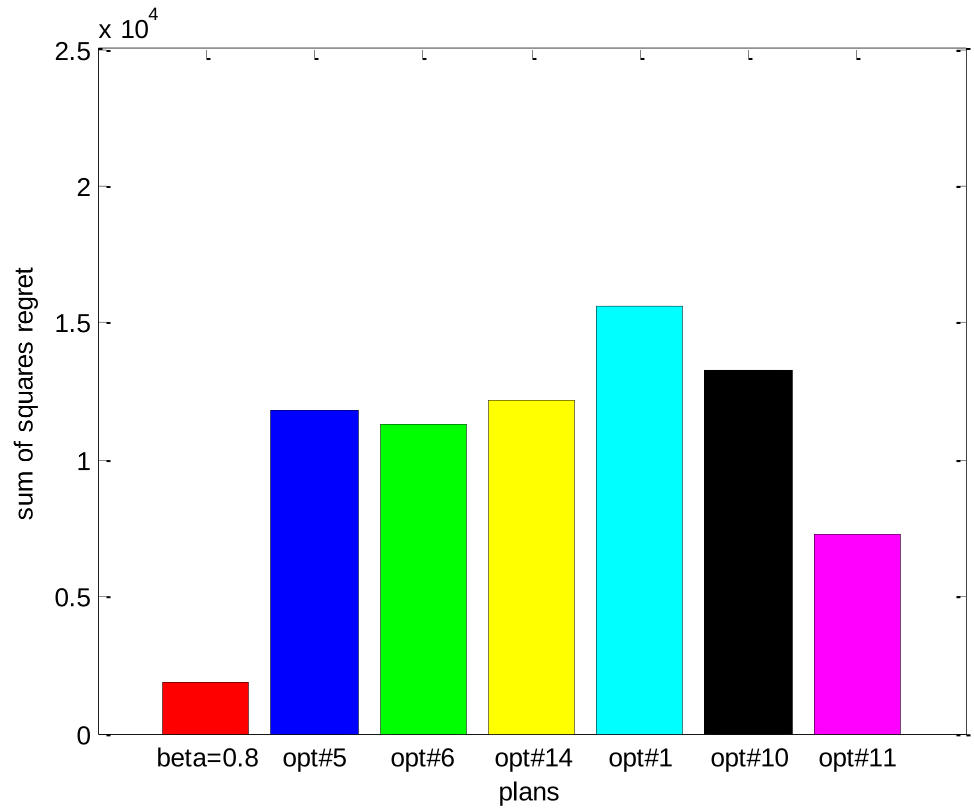

Figure 81: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 81 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- The total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the Fig.81 above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

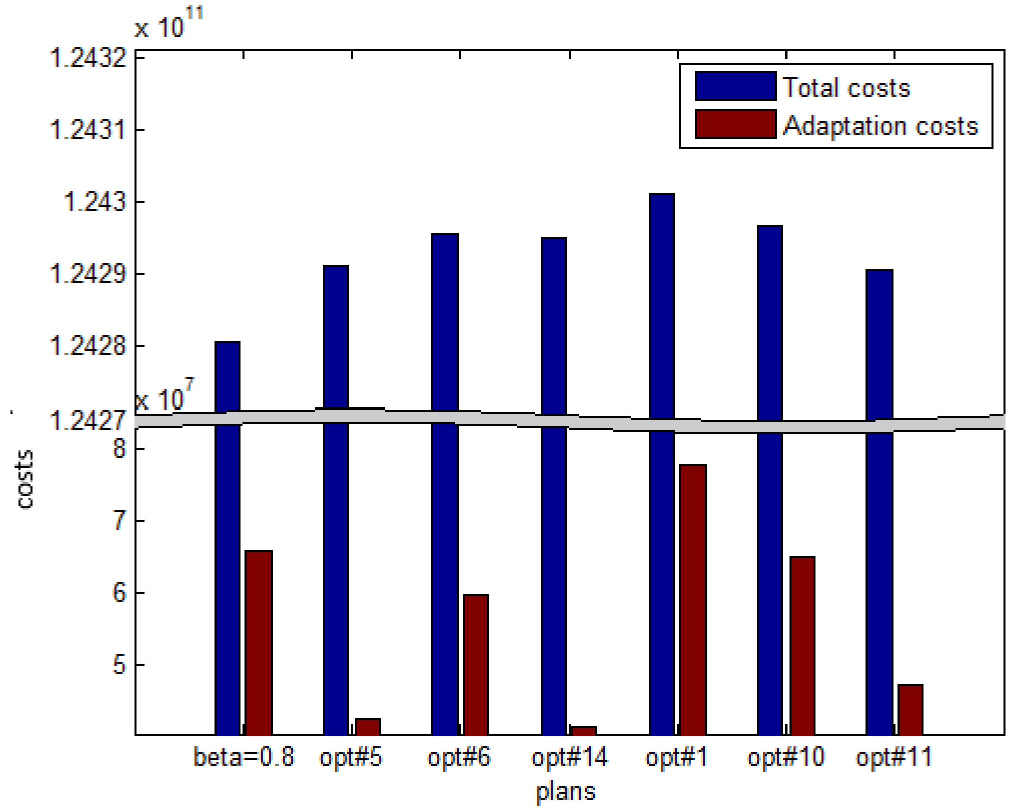

Figure 82: Average adaptation costs and total costs for different deterministic and β designs

The vertical line in Fig. 82 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig.82 that the deterministic design Opt#1 has both the highest adaptation and total costs.

Approach # 2

Figure 83 below provides the average total costs across all scenarios for the adaptation-based design (β=0.8) and different deterministic designs. Here, "total costs" refers to the sum of total investments (both the original investments (i.e. cost of core-trajectory) as well as the investments necessary to adapt to each scenario) and the total operating costs over the planning horizon. It can be seen in Fig. 83 below, that the adaptation based design has the lowest average total costs when compared to all other deterministic designs.

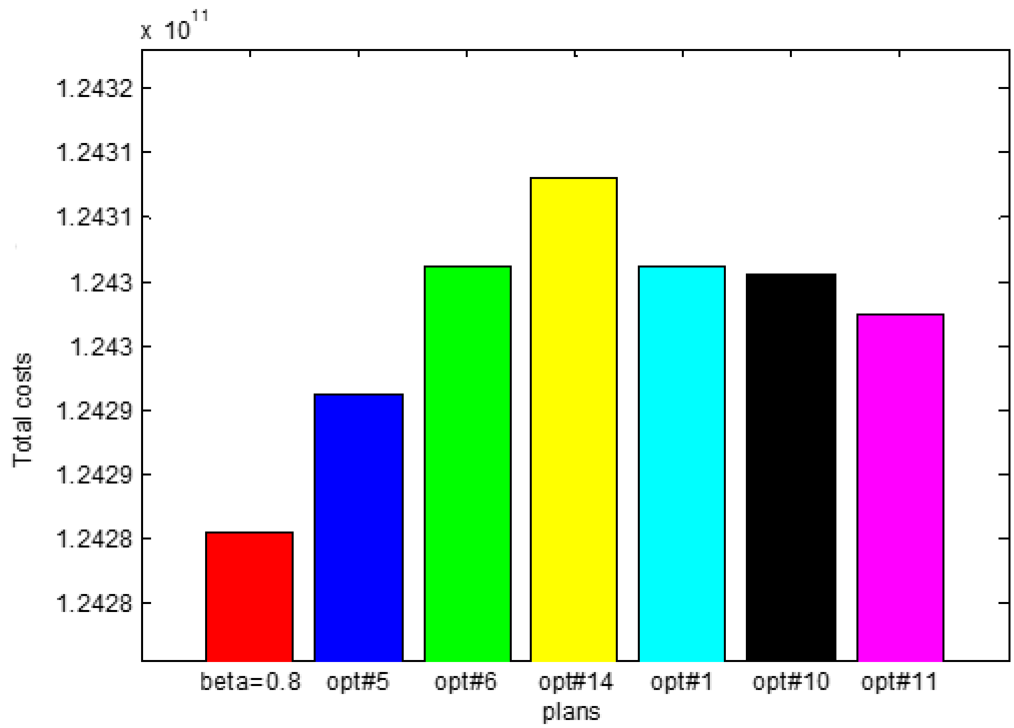

Figure 83: Average total costs across all scenarios for all scenarios for different deterministic and β designs

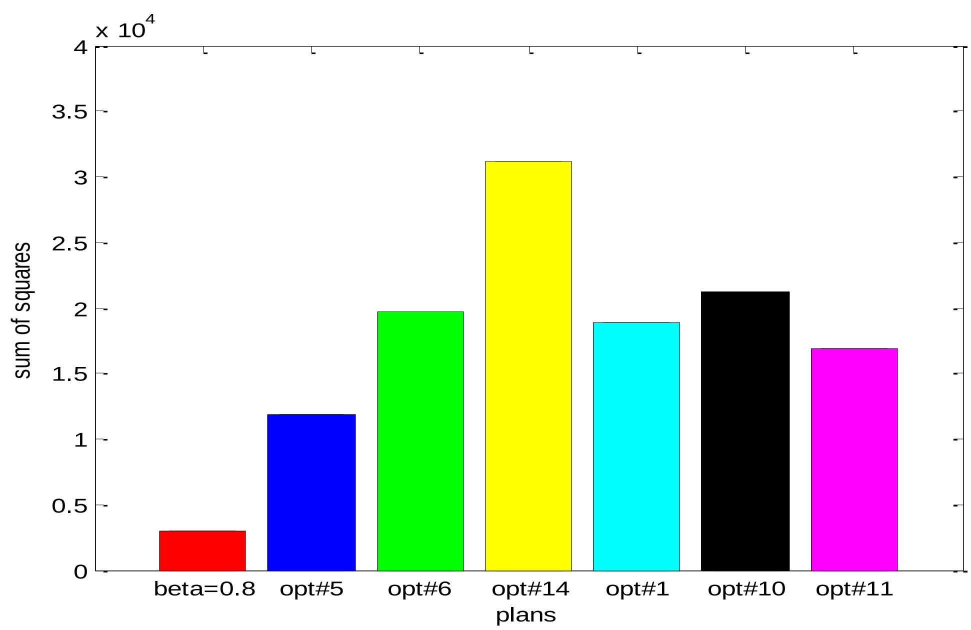

Figure 84: Sum of squares regret across all scenarios for different deterministic and β designs

To illustrate the robustness of each design, we show regret in Fig. 84 above. Here, we compute regret, for each design X, as the sum of squared differences across all scenarios between

- the total cost of design X and
- the total cost of the design having minimum total cost in the scenario.

Each of the differences are divided by a million. It can be seen in the Fig. 84 above that the adaptation based design has the lowest sum of squares regret, this confirms the consistency of adaptation based design.

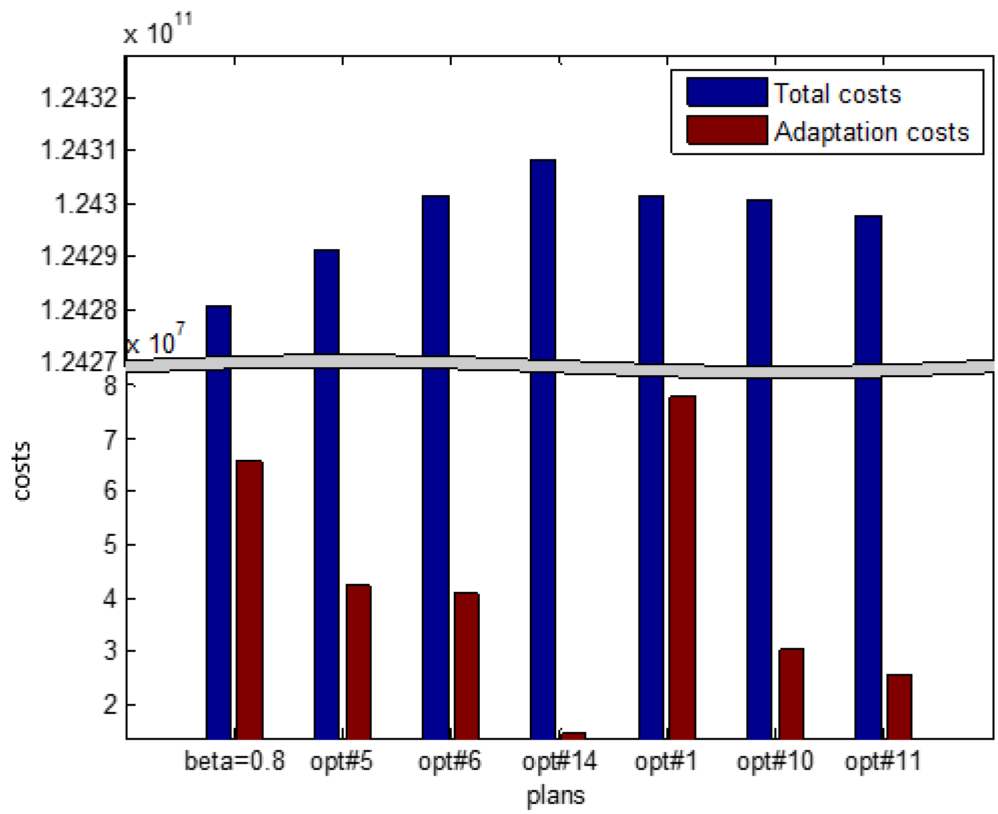

Figure 85: Average adaptation cost and total costs for different deterministic and β designs

The horizontal grey line in Fig. 85 above that passes through the bars represents a change in range, because adaptation and total costs are different in magnitude. It can be seen in Fig. 85 above that Opt#14 has the highest adaptation and total costs. β has to be well selected in order to avoid designing a robust design, robust designs tend to have low adaptation costs but very high total costs.

6.5.2 Computation issues

There are certain computational strategies that were useful for increasing the solvability of transmission expansion problems. One of them was the fact that big M has to be chosen carefully in order to avoid numerical instability [57].

Symmetry

Symmetry is a big issue in MILPs because it results in redundant computation due to solving of identical sub problems and this happens a lot in TEP with parallel lines[58]. By adding symmetry breaking inequalities such as ordering parallel candidate circuits, solution times can be significantly reduced [59]. These inequalities significantly reduced computational speed.

$$x_{ij}^{n+1} \leq x_{ij}^{n}$$

Probing

One way to deal with the computational complexity of MILP is to probe by fixing variables and adding constraints to check for the feasibility when these variables are fixed or when these constraints are added. Most commercial software such as CPLEX can detect infeasibility within seconds or few minutes. For instance if a variable is fixed to zero and the problem becomes infeasible, the variable can be set to one because the variable has be one for the optimization to be feasible. Another way is by adding a constraint to TEP problem that limits the number of lines that can be built, if by adding the constraints, the problem become infeasible, a new constraint can be added to the TEP problem that the number lines need to greater than the previous limits. This can reduce the search space of the problem.

Cplex parameter tuning

In CPLEX, parameter tuning can significantly increase the performance of an optimization problem in terms of time and performance. CPLEX allows the user to select an emphasis in for the optimization problem to focus on. The optimization problem can focus on optimality feasibility or both. Other performance parameters such as probing and symmetry and a host of others are also available.

Solving different “TEP” Parallelly

Another way to reduce computational complexity in transmission planning is to solve the optimization problem in parallel by restricting the number of lines that can be built in each sub-problem. For instance if there are 30 transmission candidates. One can solve 3 transmission planning problems by restricting the first, second and third to build lines between 0-10, 11-20 and 21-30. This will greatly reduce the combinatorial search space of the problem. As the problems are solving one can use information from their current solution to disregard the others.

Observations

One of the observation noticed when providing both lower and upper bounds for our optimization problem is that, it was better to provide a lower bound because optimization problems with lower bounds found feasible solution quicker that optimization problems with upper bounds. This is not a generalization but this was observed in this research. Another thing noticed is that the computational time for different values of βs varied. βs at the extreme ends tends to require less computational time.

CHAPTER 7. CONCLUSION AND FUTURE WORK

Deterministic transmission planning could cause a lot of regret when future scenarios differ from planned scenario. Over-reliance on a single forecast has either led to over-investment or under-investment. Limitations in deterministic transmission planning have led to the introduction of transmission planning under uncertainty. Transmission expansion planning under uncertainty is a very computational task. It is a very challenging process and a daunting task. The contributions of this work are described in the next sub-section.

7.1 Contributions

1) DESIGN OF "R2B" TRANSMISSION PLANNING UNDER UNCERTAINTY

This dissertation developed procedures for designing R2B transmission under uncertainty and applied it to the Iowa power system. A backbone is designed in order to increase the available transfer capability within and out of the state of Iowa.

2) EXTENSION OF ADAPTATION TO TRANSMISSION

One of the contributions of this dissertation is extending the adaptation approach which was originally formulated for generation expansion planning to transmission planning. After problem formulation, the approach was applied to case-studies and validated.

3) EXTENSION OF ADAPTATION TO CO-OPTIMIZATION OF TRANSMISSION AND GENERATION RESOURCES

One of the contributions of this dissertation is extending the adaptation approach which was originally formulated for generation expansion planning to co-optimization of transmission and generation resources. After problem formulation, the approach was applied to case-studies and validated.

4) IDENTIFICATION OF THE RELATIONSHIP OF STOCHASTIC PROGAMMING TO ADAPTATION

One of the contributions of this dissertation is comparing and contrasting SP and adaptation. The conceptual similarities and differences are highlighted, and formulational similarities and differences, and the treatment of uncertainty between the approaches, are also identified.

5) DEVELOPMENT OF A SCENARIO REDUCTION TECHNIQUE FOR BOTH TRANSMISSION PLANNING AND CO-OPTIMIZATION PLANNING

A scenario reduction technique was developed to reduce the computational burden associated with solving transmission expansion planning under uncertainty. The idea is to select a representative scenario that can cover a wide range of scenarios. By scenario reduction the number of constraints and variables are directly reduced, hence making the formulation more computationally tractable.

7.2 Possible Future Work

1.) MODELLING LOCAL UNCERTAINTIES

In this dissertation we model only global uncertainties due to added computational complexity involved in modelling local uncertainties. Modelling local uncertainties will improve the accuracy of adaptation-based transmission planning. Future work should consider modelling local uncertainties in the adaptation-based transmission planning model.

2.) TRANSMISSION EXPANSION PLANNING FOR THE NATIONAL GRID USING ADAPTATION

Renewable rich areas in the United States tend to have low load and most of this renewable energy will need to be transferred to high demand areas with high capacity transmission. Also depending on the location in the United States, the generation mix is very different, hence also high capacity will needed to reduce congestion, since not all regions in United States have cheap generation. The volatility of fuel price may also cause congestion at different times. A future continuation of this work could be applying the adaptation approach to transmission planning under uncertainty at the national level.

3.) CO-OPTIMIZATION FOR THE NATIONAL GRID USING ADAPTATION

The generation mix at different regions in the United States is changing. This is due to change in government policies and other unpredictable uncertainties. This change is likely to affect the change of the flow of power on the national grid level. A co-optimization formulation under uncertainty will likely co-ordinate the change of generation mix and the transmission lines needed to be built to facilate the change. A future continuation of this work could be applying the adaptation approach to co-optimization under uncertainty at the national level. A co-optimization formulation at the national level will help policy maker's long term decisions for the United States.

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APPENDIX IEEE 24-BUS SYSTEM DATA

Table33: Branch data (all per-unit data given on a 100 MVA base)

From	To	Reactance	Capacity (MW)
1	2	0.001231	175
1	3	0.211137	175
1	5	0.084901	175
2	4	0.1273	175
2	6	0.192014	175
3	9	0.11921	175
3	24	0.0839	400
4	9	0.107787	175
5	10	0.088372	175
6	10	0.033329	175
7	8	0.06103	175
8	9	0.164817	175
8	10	0.164817	175
9	11	0.0839	400
9	12	0.0839	400
10	11	0.0839	400
10	12	0.0839	400
11	13	0.04746	500
11	14	0.041725	500
12	13	0.0476	500
12	23	0.096441	500
13	23	0.086428	500
14	16	0.038841	500
15	16	0.17269	500
15	21	0.04899	500
15	21	0.04899	500
15	24	0.051867	500
16	17	0.025901	500
16	19	0.023024	500
17	18	0.014392	500
17	22	0.105266	500
18	21	0.025951	500
18	21	0.025951	500
19	20	0.039636	500
19	20	0.039636	500
20	23	0.021627	500
20	23	0.021627	500
21	12	0.06769	500

Table34: Transmission line candidates (all per-unit data given on a 100 MVA base)

From	To	Reactance	Capacity (MW)
1	2	0.001231	175
1	3	0.211137	175
1	5	0.084901	175
2	4	0.1273	175
2	6	0.192014	175
3	9	0.11921	175
3	24	0.0839	400
4	9	0.107787	175
5	10	0.088372	175
6	10	0.033329	175
7	8	0.06103	175
8	9	0.164817	175
8	10	0.164817	175
9	11	0.0839	400
9	12	0.0839	400
10	11	0.0839	400
10	12	0.0839	400
11	13	0.04746	500
11	14	0.041725	500
12	13	0.0476	500
12	23	0.096441	500
13	23	0.086428	500
14	16	0.038841	500
15	16	0.17269	500
15	21	0.04899	500
15	21	0.04899	500
15	24	0.051867	500
16	17	0.025901	500
16	19	0.023024	500
17	18	0.014392	500
17	22	0.105266	500
18	21	0.025951	500
18	21	0.025951	500
19	20	0.039636	500
19	20	0.039636	500
20	23	0.021627	500
20	23	0.021627	500
21	12	0.06769	500

Table 35: Generator data for transmission expansion planning case-study

Bus number	Capacity	Generator type
1	469.33MW	Nuclear
2	469.33MW	NG
7	733.33MW	Coal
13	1444.66MW	NG
15	525.56MW	Wind
16	378.88MW	NG
18	977.77MW	Nuclear
21	977.77MW	Coal
22	733.33MW	Nuclear
23	1613.33MW	NG

Table 36: Load data

Load bus	Load ratio *
1	0.0379
2	0.034
3	0.0632
4	0.026
5	0.0249
6	0.0477
7	0.0439
8	0.06
9	0.0614
10	0.0684
13	0.093
14	0.0681
15	0.1112
16	0.0351
21	0.1168
22	0.0635
23	0.0449

*By the load ratio is the ratio of load at a given bus to the total load of the system.

Table 37 :Generator data for co-optimization expansion planning case-study

Bus number	Capacity	Generator type
1	287.99MW	Nuclear
2	287.99MW	NG
7	449.99MW	Coal
13	886.45MW	NG
15	322.50MW	Wind
16	232.49MW	NG
18	599.99MW	Nuclear
21	599.99MW	Coal
22	449.99MW	Nuclear
23	989.99MW	NG